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Beyond the Standard Model

GUT's 2012

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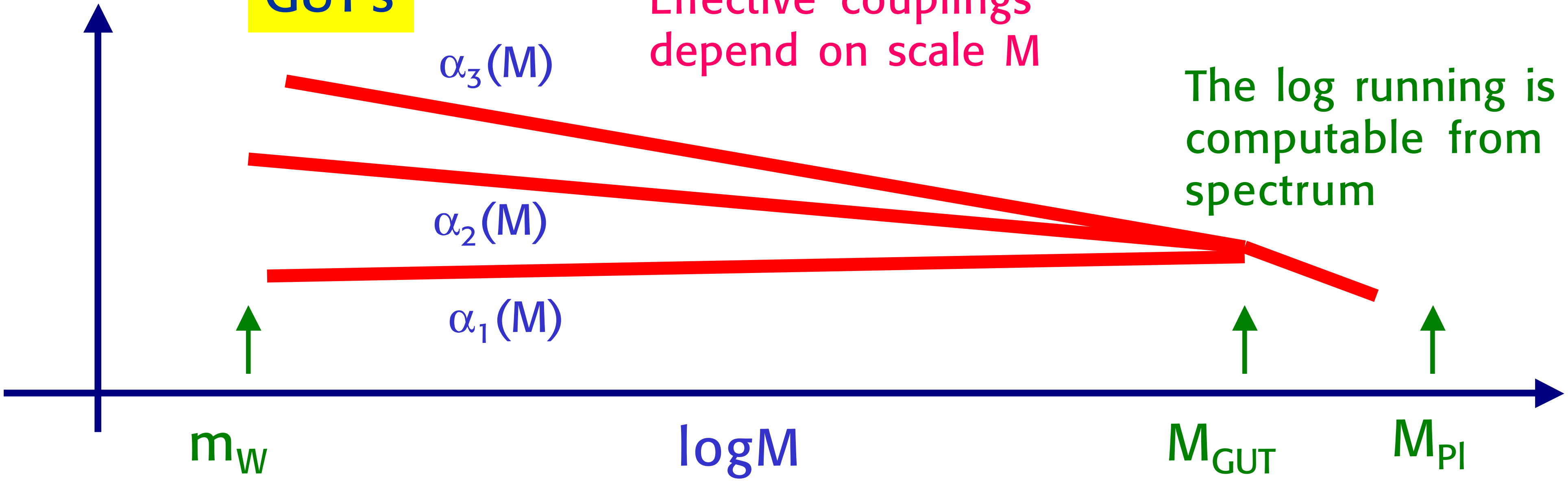
The programme of Grand Unification

- At a large scale M_{GUT} the gauge symmetry is extended to a group G:

$$G \supset SU(3) \otimes SU(2) \otimes U(1)$$

- G is spont. broken and the additional generators correspond to heavy gauge bosons with masses m ~ M_{GUT}
- At M_{GUT} there is a single gauge coupling
- The differences of couplings at low energies are due to the running from M_{GUT} down to m_Z
- The observed SM charges of quark and leptons are determined by the representations of G

GUT's



The large scale structure of particle physics:

- SU(3) ⊗ SU(2) ⊗ U(1) unify at M_{GUT}
- at $M_{Pl} \sim 10^{19}$ GeV: quantum gravity

$$G_{Newton} = \frac{\hbar c}{M_{Pl}^2}$$

$r \sim 10^{-33}$ cm

Superstring theory (?):
a 10-dimensional non-local, unified theory of all interact's

The really fundamental level

By now GUT's are part of our culture in particle physics

- Unity of forces: $G \supset SU(3) \otimes SU(2) \otimes U(1)$

  unification of couplings
- Unity of quarks and leptons

  different "directions" in G
- B and L non conservation

  -> p-decay, baryogenesis, ν masses
- Family Q-numbers

  e.g. in SO(10) a whole family in 16
- Charge quantization: $Q_d = -1/3 \to -1/N_{colour}$

  anomaly cancelation

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Most of us believe that Grand Unification must be a feature of the final theory!

$$G \supset SU(3) \otimes SU(2) \otimes U(1)$$

G commutes with the Poincare' group

→ repres.ns must contain states with same momentum, spin..

We cannot use e^-_L, e^-_R, but need all L or all R

$$e^-_R \xrightarrow{\text{CPT}} e^+_L$$

We can use e^-_L, e^+_L etc. One family becomes

$$3 \text{ x } \begin{bmatrix} u \\ d \end{bmatrix}_L \qquad \begin{bmatrix} \nu \\ e^- \end{bmatrix}_L \qquad \begin{matrix} 3 \text{ x } u^{bar}_L \\ 3 \text{ x } d^{bar}_L \end{matrix} \qquad e^+_L \qquad (\nu^{bar}_L)$$

Note that in each family there are 15 (16) two-component spinors

SU(5): 5^{bar} + 10 + (1)

SO(10): 16

Group Theory Preliminaries

Gauge group: $U = \exp(ig\Sigma_A \theta^A T^A)$

g: gauge coupling, $\theta^A = \theta^A(x)$: parameters,

T^A: basis of generators, A=1,...., D

If U is a unitary matrix, T^A are hermitian (e.g. SU(N))

$[T^A,T^B]=iC^{ABC}T^C$ C^{ABC}: structure constant

In a given N-dim repres.n of G: T^A -> t^A with t^A a NxN matrix.

The normalisation of T^A is fixed if we take $Tr(t^A t^B)=1/2\ \delta^{AB}$ in some simplest repres'n (e.g the N in SU(N), fundamental repres'n)

This also fixes the norm'n of C_{ABC} and g

If $Tr(t^A t^B)=1/2\ \delta^{AB}$ then $Tr(t'^A t'^B)=c\ \delta^{AB}$
in another repres'n, with c=constant

General requirements on G

The rank of a group is the maximum number of generators that can be simultaneously diagonalised

SU(N) (group of NxN unitary matrices with det=1) has rank N-1

SU(3)XSU(2)XU(1) has rank 2+1+1=4

SU(N) transf: $U = \exp(i\Sigma_A \theta^A t^A)$ A=1,...,N^2-1

Recall: if U =exp(iT), then det U = exp(iTrT).
In fact both det and tr are invariant under diagonalisation.
So detU=1 -> trT=0.

The group G must have rank r≥4 and admit complex repres.ns (e.g. quarks and antiquarks are different)

r=4: SU(5), SU(3)xSU(3) → (actually does not work)
→ (+ discrete simmetry)
r=5: SO(10)
r=6: E6, SU(3)xSU(3)xSU(3) →

For products like SU(3)xSU(3) or SU(3)xSU(3)xSU(3) a discrete symmetry that interchanges the factors is also undestood so that the gauge couplings are forced to be equal.

The particle content must also be symmetric under the same interchange:

For example, in SU(3)xSU(3)xSU(3):

$$(3,3^{\mathrm{bar}},1) + (3^{\mathrm{bar}},1,3) + (1,3,3^{\mathrm{bar}})$$

q	anti-q	leptons

SU(5) Representations

The embedding of 3x2x1 into 5 is specified once we give the content of the fundamental representation 5.

Q

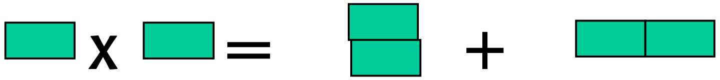
$$5 = (3,1)_{-1/3}+(1,2)_{1,0}$$

colour E-W

$$\bar{5} = (\bar{3},1)_{1/3}+(1,2)_{0,-1}$$

$$\bar{5} = \begin{bmatrix}\bar{d}_1\\ \bar{d}_2\\ \bar{d}_3\\ \nu\\ e^-\end{bmatrix}$$

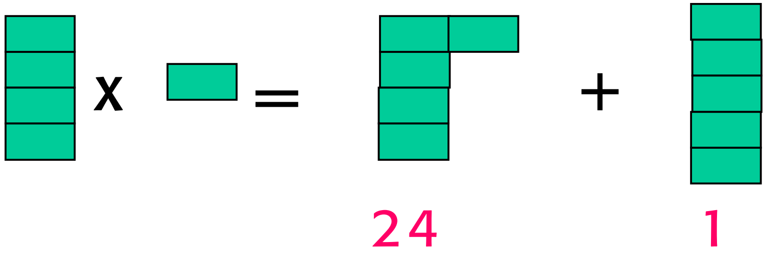
5x5antisymm.= $(\bar{3},1)_{-2/3}+(3,2)_{2/3,-1/3}+(1,1)_1 = 10$



5x5symm.= $(6,1)_{-2/3}+(3,2)_{2/3,-1/3}+(1,3)_{2,1,0} = 15$

g B W

$\bar{5}$x5= $(8,1)_0+(1,1)_0+(3,2)_{-1/3,-4/3}+(\bar{3},2)_{1/3,4/3}+(1,3)_{1,0,-1}+(1,1)_0 = 24+1$



Content of SU(5) representations (apart from phases)

$$\bar{5} = \begin{bmatrix} \bar{d}_1 \\ \bar{d}_2 \\ \bar{d}_3 \\ \nu \\ e^- \end{bmatrix}$$

$$10 = \begin{bmatrix} 0 & \bar{u}_3 & \bar{u}_2 & u_1 & d_1 \\ - & 0 & \bar{u}_1 & u_2 & d_2 \\ - & - & 0 & u_3 & d_3 \\ - & - & - & 0 & e^+ \\ - & - & - & - & 0 \end{bmatrix}$$

$$24 = \begin{bmatrix} g & g & g & X_1^{4/3} & Y_1^{1/3} \\ g & g & g & X_2^{4/3} & Y_2^{1/3} \\ g & g & g & X_3^{4/3} & Y_3^{1/3} \\ X_1^{-4/3} & X_2^{-4/3} & X_3^{-4/3} & W^3 & W^+ \\ Y_1^{-1/3} & Y_2^{-1/3} & Y_3^{-1/3} & W^- & B \end{bmatrix}$$

SU(5) breaking

Simplest possibility:

24
SU(5) ---> SU(3)xSU(2)xU(1)

$$\langle \Sigma \rangle = M \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & \frac{-3}{2} & 0 \\ 0 & 0 & 0 & 0 & \frac{-3}{2} \end{bmatrix}$$

The Doublet -Triplet Splitting Problem

In SU(5) the mass terms in the Higgs sector are

$W=aH_{5Bar}\Sigma_{24}H_5+mH_{5Bar}H_5$

$$H_5 = \begin{bmatrix} H_{T1} \\ H_{T2} \\ H_{T3} \\ H^+ \\ H^0 \end{bmatrix} \begin{matrix} \left.\begin{matrix} \\ \\ \\ \end{matrix}\right\} \text{colour triplet} \xrightarrow{\qquad} \\ \left.\begin{matrix} \\ \\ \end{matrix}\right\} \text{usual doublet} \end{matrix}$$

$$\langle \Sigma \rangle = M \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & \frac{-3}{2} & 0 \\ 0 & 0 & 0 & 0 & \frac{-3}{2} \end{bmatrix}$$

Higgs masses:

$m_{HT} = + aM +m$

$m_H = -3/2\ aM +m \sim 0$

Since $M \sim m \sim M_{GUT}$ it takes an enormous fine-tuning to set m_H to zero.

SUSY slightly better because once put by hand at tree level is not renormalised.

Is a big problem for minimal models (see later)

SO(N)

NxN orthogonal matrices with det=1

$R^TR=RR^T=1$

for small ε: $R(\varepsilon)_{ab}=\delta_{ab}+\varepsilon_{ab}$

$\varepsilon+\varepsilon^T=0$

$\varepsilon_{ab}=-\varepsilon_{ba}$

$R(\theta)=\exp[i\theta^{AB}T^{AB}/2]$

T antisymmetric, imaginary

#generators=# antisymm. matrices

$D=N(N-1)/2$ [$D=45$ for SO(10)]

Imposing that for infinitesimal transf.: $\varepsilon_{ab}=i\varepsilon_{AB}(T_{AB})_{ab}/2$

one finds: $(T_{AB})_{ab}=-i(\delta_{Aa}\delta_{Bb}-\delta_{Ba}\delta_{Ab})$ --> $\mathrm{Tr}T_{AB}=0$

$$[T_{AB},T_{CD}]=-i[\delta_{BC}T_{AD}+\delta_{AD}T_{BC}-\delta_{AC}T_{BD}-\delta_{BD}T_{AC}]$$

If A, B, C, D are all different $[T_{AB},T_{CD}]=0$.

For SO(10) $T_{12}, T_{34}, T_{56}, T_{78}, T_{910}$ all commute: SO(10) has rank 5

SO(N) has rank N/2 or (N-1)/2 for N even or odd.

“Orbital” real representations of SO(10)

10 is the fundamental, 45 is the adjoint

10x10 = 54 + 45 + 1   45 is antisymm, 54 and 1 are symm

In addition to orbital repres’ns SO(2N) also has spinorial representations (recall SO(3) <-> SU(2) relation).

Γ_μ (μ=1,2,....,2N) are 2^Nx2^N matrices satisfying

$\{\Gamma_\mu,\Gamma_\nu\} = 2\delta_{\mu\nu}$ (implies $\Gamma_\mu{}^2=1$ and Tr $\Gamma_\mu=0$); $\Gamma_\mu{}^+ = \Gamma_\mu$

Then $\Sigma_{\mu\nu}$ obey the group commutator algebra, where

$\Sigma_{\mu\nu} = \frac{i}{4}[\Gamma_\mu,\Gamma_\nu]$ and $S(\theta)=\exp[i\theta_{\mu\nu}\Sigma_{\mu\nu}/2]$ is a spinorial repres’n

Γ_μ can be written down in the form (σ_j are Pauli matrices):

$$\Gamma_{2i} = \underbrace{1 \otimes 1 \otimes \ldots. \otimes 1}_{\text{i-1}} \otimes \sigma_1 \otimes \underbrace{\sigma_3 \otimes \sigma_3 \otimes \ldots. \otimes \sigma_3}_{\text{N-i}}$$

$$\Gamma_{2i-1} = 1 \otimes 1 \otimes \ldots. \otimes 1 \otimes \sigma_2 \otimes \sigma_3 \otimes \sigma_3 \otimes \ldots. \otimes \sigma_3$$

$S(\theta)=\exp[i\theta_{\mu\nu}\Sigma_{\mu\nu}/2]$ acts on a 2^N-dimensional spinor ψ:

$\psi' = S\psi$ One has for $\theta=\varepsilon$ infinitesimal: $S^+\Gamma_\mu S \sim \Gamma_\mu + \varepsilon_{\nu\mu}\Gamma_\nu$

or, in general: $S^+\Gamma_\mu S = R_{\mu\nu}{}^T\Gamma_\nu$

There is a chiral operator $\Gamma_0=(i)^N\Gamma_1\Gamma_2....\Gamma_{2N}$ (analogous to γ_5)

$\Gamma_0{}^2=1$, Tr $\Gamma_0=0$, $\Gamma_0{}^+=\Gamma_0$, $\{\Gamma_0,\Gamma_\mu\}=0$, $[\Gamma_0,\Sigma_{\mu\nu}]=0$

$$\Gamma_0=\sigma_3\otimes\sigma_3\otimes........\otimes\sigma_3\otimes\sigma_3\otimes\sigma_3$$

Thus Γ_0 commutes with the generators and has eigenvalues ±1: the spinorial representation splits into 2 halves.

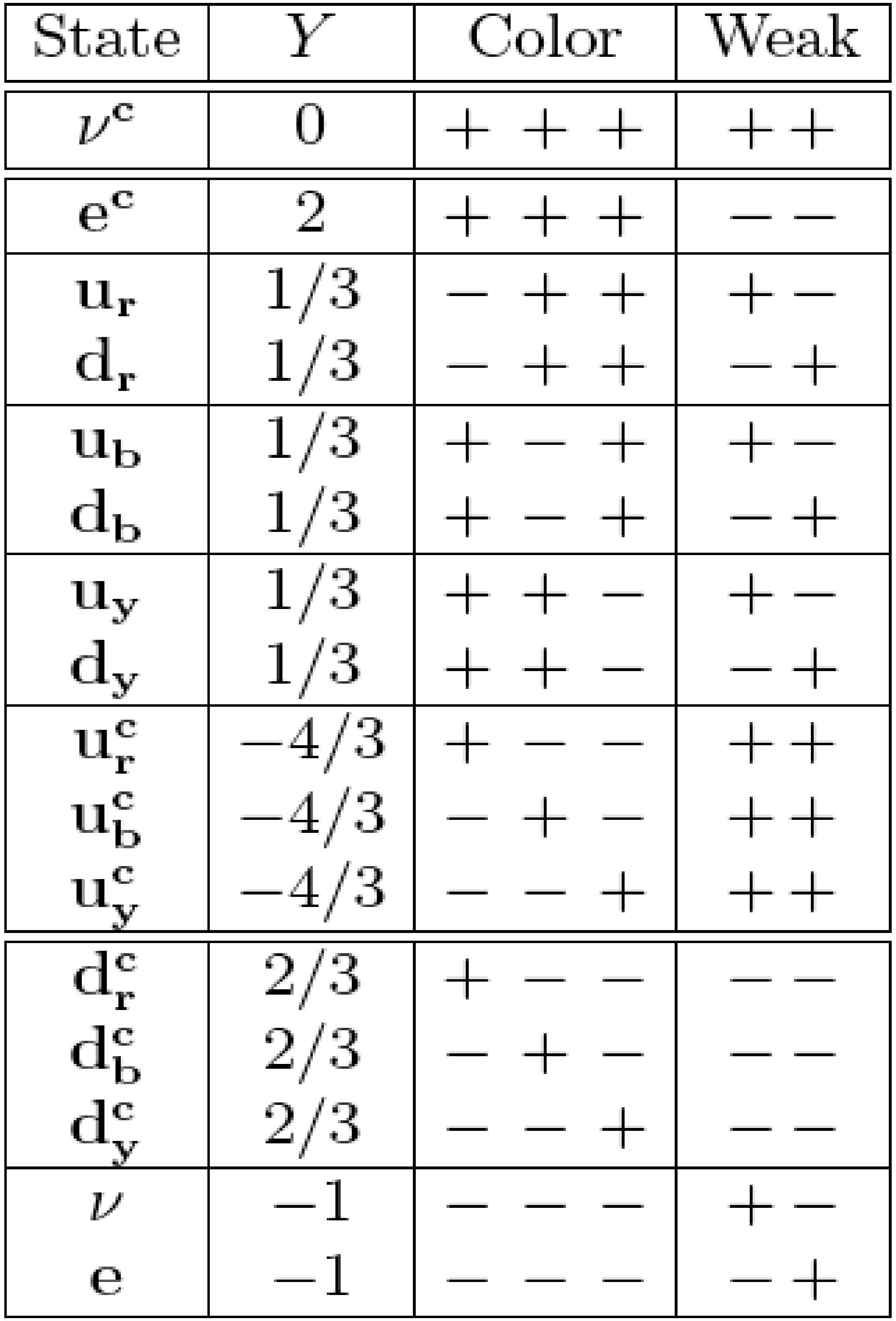
In SO(10) $32 = 16 + 16^{bar}$

$16x16 = 10 + 126 + 120$

$16x16^{bar} = 1 + 45 + 210$

$$\begin{array}{ccccccccc} \psi^+\{ & 1, & \Gamma_\mu, & \Gamma_\mu\Gamma_\nu, & \Gamma_\mu\Gamma_\nu\Gamma_\lambda, & \Gamma_\mu\Gamma_\nu\Gamma_\lambda\Gamma_\rho, & \Gamma_\mu\Gamma_\nu\Gamma_\lambda\Gamma_\rho\Gamma_\sigma & \}\psi \\ & 1 & 10 & 45 & 120 & 210 & 126 & \end{array}$$

The 16 of SO(10)
can be generated
by 5 spin 1/2
with even number of
$s_3 = -1/2$



State	Y	Color	Weak
ν^c	0	$+ + +$	$+ +$
e^c	2	$+ + +$	$- -$
u_r	1/3	$- + +$	$+ -$
d_r	1/3	$- + +$	$- +$
u_b	1/3	$+ - +$	$+ -$
d_b	1/3	$+ - +$	$- +$
u_y	1/3	$+ + -$	$+ -$
d_y	1/3	$+ + -$	$- +$
u^c_r	$-4/3$	$+ - -$	$+ +$
u^c_b	$-4/3$	$- + -$	$+ +$
u^c_y	$-4/3$	$- - +$	$+ +$
d^c_r	2/3	$+ - -$	$- -$
d^c_b	2/3	$- + -$	$- -$
d^c_y	2/3	$- - +$	$- -$
ν	-1	$- - -$	$+ -$
e	-1	$- - -$	$- +$

1

10

5^{bar}

SO(10) Multiplication Table

(s means "symmetric")

10x10= 1s+45 +54s
10x16= 16bar+144
16x16bar=1+45+210
16x16=10s+120+126s
10x45= 10+120+320
16x45= 16+144bar+560
45x45= 1s+45+54s+210s+770s+945
10x120= 45+210+945

SO(10) is very impressive

A whole family in a single representation 16

$$\underset{SO(10)}{\mathbf{16}} \supset \underset{SU(5)}{\bar{\mathbf{5}}} + \mathbf{10} + \mathbf{1} \longleftarrow \nu_R$$

Too striking not to be a sign! SO(10) must be relevant at least as a classification group.

Different avenues for SO(10) breaking:

We could have [SO(10) contains SU(5)xU(1)]:

$$SO(10) \xrightarrow[M_{Pl}]{16} SU(5) \xrightarrow[M_{GUT}]{45} SU(3)\times SU(2)\times U(1)$$

and SU(5) physics is completely preserved

or $\longrightarrow$

Other interesting subgroups of SO(10) are

$$SO(10) \supset SU(4)_{PS} \otimes SU(2)_L \otimes SU(2)_R \qquad 54$$

$$SO(10) \supset SU(3) \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L} \qquad 45$$

10x10=1+45+54 →

These breakings can occur anywhere from M_{GUT} down.
Possibility of two steps: M_{GUT} -> $M_{intermediate}$ -> M_{weak}.
In this case with $M_{intermediate}$ ~ 10^{11}-^{12} GeV good coupling unification without SUSY.

PS= Pati-Salam: L as the 4th colour

16: $\begin{bmatrix} u & u & u & \nu \\ d & d & d & e \end{bmatrix}_L = (4, 2, 1)$ $\qquad \begin{bmatrix} u & u & u & \nu \\ d & d & d & e \end{bmatrix}^{bar}_R = (4, 1, 2)^{bar}$

Also note: $Q=T^3_L+T^3_R+(B-L)/2$

Left-Right symmetry (parity) is broken spontaneously

In SM the covariant derivative is:

$$D_\mu = \partial_\mu + ie_s \sum_{c=1}^{8} t^c g^c{}_\mu + ig \sum_{i=1}^{3} t^i W^i{}_\mu + ig' \frac{Y}{2} B_\mu$$

$t^c = \frac{\lambda^c}{2}$ Gell-Mann $t^i = \frac{\tau^i}{2}$ Pauli

$Tr(t^c t^{c'}) = 1/2\ \delta^{cc'}$ $Tr(t^i t^{i'}) = 1/2\ \delta^{ii'}$

$$\alpha_s \equiv \alpha_3 = \frac{e_s^2}{4\pi} \qquad \alpha_W \equiv \alpha_2 = \frac{g^2}{4\pi} \qquad \alpha_1 = \frac{g'^2}{4\pi}$$

In G gauge th. the covariant derivative is:

$$D_\mu = \partial_\mu + ig_G \sum_{A=1}^{d} T^A X^A{}_\mu$$

g_G: symm. coupl.

X^A: G gauge bos'ns $Tr(T^A T^B) \sim \delta^{AB}$

I can always choose the T^A norm'n as:

$Q=t^3+Y/2$ ⟶ $Q=T^3+bT^0$ Then $aT^c=\lambda^c/2$

a,b: const's dep. on G and the 3x2x1 embedding

The G-symmetric cov. derivative contains:

$$g_G \sum T^c g^c_\mu + g_G \sum T^i W^i_\mu + g_G T^0 B_\mu$$

or

$$\frac{g_G}{a} \sum t^c g^c_\mu + g_G \sum t^i W^i_\mu + \frac{g_G}{b}\frac{Y}{2} B_\mu$$

comparing with:

$$D_\mu = \partial_\mu - i e_s \sum_{c=1}^{8} t^c g_\mu{}^c - i g \sum_{i=1}^{3} t^i W_\mu{}^i - i g' \frac{Y}{2} B_\mu$$

we find: $\alpha_G = \frac{g_G^2}{4\pi}$ ← the one which is unified

$$\alpha_s \equiv \alpha_3 = \frac{\alpha_G}{a^2} \qquad \alpha_W \equiv \alpha_2 = \alpha_G \qquad \alpha_1 = \frac{\alpha_G}{b^2}$$

$\mathrm{tg}^2\theta_W = \alpha_1/\alpha_2 = 1/b^2$ $\sin^2\theta_W = 1/(1+b^2)$

From $Q=T^3+bT^0$ we find:

$Tr(T^AT^B) \sim \delta^{AB}$

$TrQ^2= (1+b^2)trT^2$

$tr(T^3)^2= tr(T^0)^2= tr(T^A)^2 =trT^2$

From $aT^c=\lambda^c/2$ we have:

$a^2TrT^2=Tr(\lambda^c/2\ )^2$

tr is over any red. or irred. repr. of G

IF all particles in one family fill one such repres. of G:

$3 \text{ x } \begin{bmatrix} u \\ d \end{bmatrix}_L$ $\begin{bmatrix} \nu \\ e^- \end{bmatrix}_L$ $3 \text{ x } u^{bar}{}_L$ $3 \text{ x } d^{bar}{}_L$ $e^+{}_L$ $(\nu^{bar}{}_L)$

$$Tr(T^3)^2 = 3 \cdot \left(\frac{1}{4}+\frac{1}{4}\right) + \left(\frac{1}{4}+\frac{1}{4}\right) = 2$$

$$TrQ^2 = (3+3) \cdot \left(\frac{4}{9}+\frac{1}{9}\right) + 1 + 1 = \frac{16}{3}$$

$$Tr\left(\frac{\lambda_3}{2}\right)^2 = (2+2) \cdot \left(\frac{1}{4}+\frac{1}{4}\right) = 2$$

$\longrightarrow$ $b^2=5/3,\ \ a^2 = 1$

(SUSY) GUT's: Coupling Unification at 1-loop

SU(5), SO(10)
b^2=5/3
a=1

$$\frac{1}{b^2\alpha_1(\mu)} = \frac{1}{\alpha_G(M)} - \beta_1 \ln\frac{M^2}{\mu^2}$$

$$\frac{1}{\alpha_2(\mu)} = \frac{1}{\alpha_G(M)} - \beta_2 \ln\frac{M^2}{\mu^2}$$

$$\frac{1}{a^2\alpha_3(\mu)} = \frac{1}{\alpha_G(M)} - \beta_3 \ln\frac{M^2}{\mu^2}$$

SM

$$\beta_1 = -\frac{3}{5}\cdot\frac{n_H}{24\pi} + X$$

$$\beta_2 = \frac{11\cdot 2}{12\pi} - \frac{n_H}{24\pi} + X$$

$$\beta_3 = \frac{11\cdot 3}{12\pi} + X$$

SUSY

$$\beta_1 = -\frac{3}{5}\cdot\frac{3n_H}{24\pi} + X$$

$$\beta_2 = \frac{18}{12\pi} - \frac{3n_H}{24\pi} + X$$

$$\beta_3 = \frac{27}{12\pi} + X$$

We take as independent variables

$$(\sin\theta_W)^2 \equiv s_W^2 \quad , \quad \alpha \quad , \quad \alpha_3$$

In terms of them:

$$\alpha_2 = \frac{\alpha}{s_W^2} \qquad \alpha_1 = \frac{\alpha}{c_W^2}$$

From (here $\alpha=\alpha(\mu)$)

$$\frac{1}{b^2\alpha_1} - \frac{1}{\alpha_2} = (\beta_2 - \beta_1)\cdot \ln\frac{M^2}{\mu^2}$$

$$\frac{1}{\alpha_2} - \frac{1}{a^2\alpha_3} = (\beta_3 - \beta_2)\cdot \ln\frac{M^2}{\mu^2}$$

For m=μ the differences vanish e.g. →

$$s_W^2\Big|_{at\ M} = \frac{1}{1+b^2}$$

Setting b^2=5/3 and a=1and n_H =2 in SUSY:

$$\frac{7}{5}\cdot\left(\frac{3}{5}\cdot\frac{c_W^2}{\alpha} - \frac{s_W^2}{\alpha}\right) = \frac{1}{\pi}\ln\frac{M^2}{\mu^2} = \frac{s_W^2}{\alpha} - \frac{1}{\alpha_3}$$

Equivalently:

$$s_W^2 = \frac{7}{15}\cdot\frac{\alpha}{\alpha_3} + \frac{1}{5} \qquad \ln\frac{M}{\mu} = \frac{\pi}{10}\cdot\left(\frac{1}{\alpha} - \frac{8}{3}\cdot\frac{1}{\alpha_3}\right)$$

1-loop SUSY:

$$s_W^2 = \frac{7}{15} \cdot \frac{\alpha}{\alpha_3} + \frac{1}{5} \qquad \ln \frac{M}{\mu} = \frac{\pi}{10} \cdot \left(\frac{1}{\alpha} - \frac{8}{3} \cdot \frac{1}{\alpha_3} \right)$$

Suppose we take $\mu \sim 100$ GeV, $s_W{}^2 \sim 0.23$, $\alpha \sim 1/129$
we obtain $\alpha_3 \sim 0.12$. The measured value at μ is just about 0.12.
(in the SM we would have obtained $\alpha_3 \sim 0.07$)

From the second eq. with $\alpha_3 \sim 0.12$ we find
$M \sim 4\ 10^{16}$ GeV (in SM $M \sim 2\ 10^{15}$ GeV).

From this simple 1-loop approx. we see that SUSY is much better than SM for both unification and p-decay
(p-decay rate scales as M^{-4}).

We now refine the evaluation by taking 2-loop beta functions and threshold corrections into account.

In the SUSY case there is a lot of sensitivity on the number of H doublets ($n_H=2+\delta$)

$$\alpha_3 = \alpha \cdot \frac{56-2\delta}{s_W^2 \cdot (120+6\delta)-(24+3\delta)}$$

δ	n_H	α_3
-2	0	0.068
-1	1	0.086
0	2	0.121
1	3	0.211
2	4	1.120

α_3-> infinity for δ=2.22...

So just 2 doublets are needed in SUSY and this is what is required in the MSSM!

In SM we would need n_H~7 to approach α_3~0.12

The value of $\alpha_3(\mu)$ for unification, given s^2_W and α, is modified as:

1-loop

$$\alpha_3 = \frac{\alpha_3^{LO}}{1 + \alpha_3^{LO}\delta} \qquad \delta = k + \frac{1}{2\pi}\log\frac{m_{SUSY}}{m_Z} - \frac{3}{5\pi}\log\frac{m_{H_T}}{m_{GUT}^{LO}}$$

$$k = k_2 + \underbrace{k_{SUSY} + k_{GUT}}_{\text{thresholds}}$$

2-loop

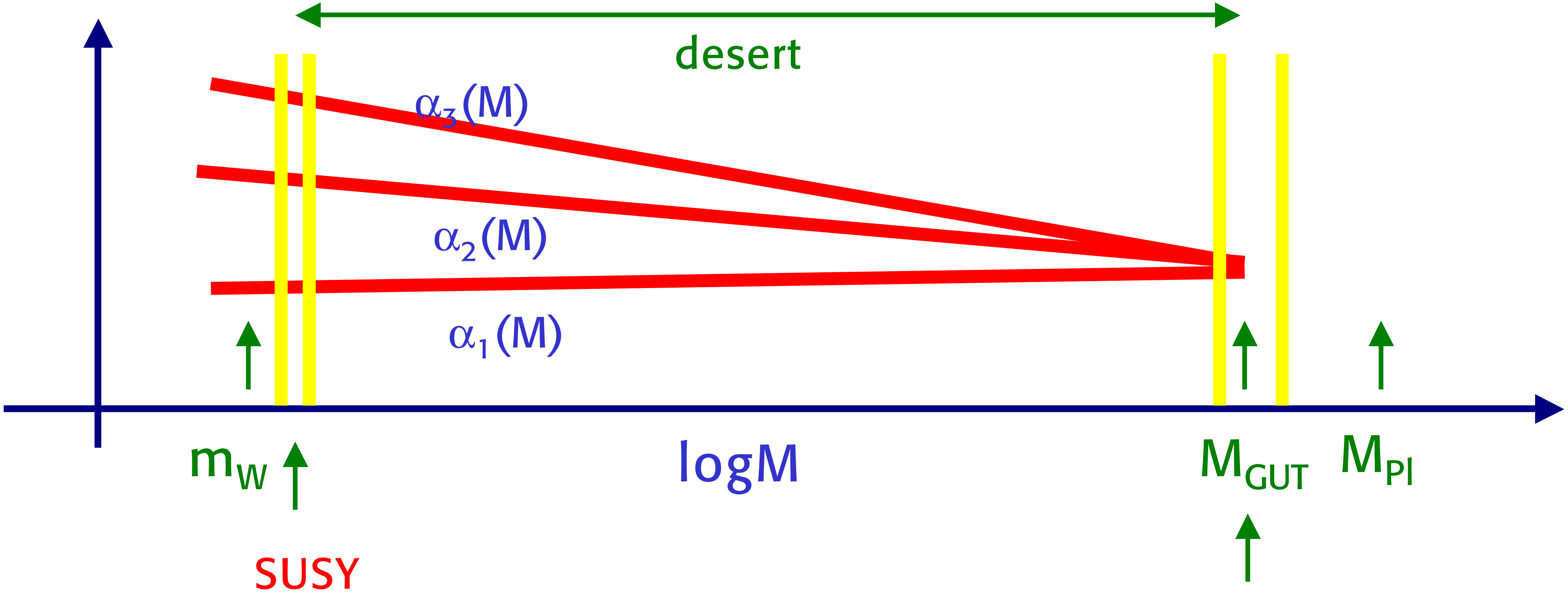
k_2 ~ -0.733

k_{SUSY} describes the onset of the SUSY threshold at around m_{SUSY}

k_{GUT} describes effects of the splittings inside (in SU(5)) the 24, 5 and 5^{bar}

Beyond leading approx. we define m_{GUT} as the mass of the heavy 24 gauge bosons, while $m_T = m_{HT}$ is the mass of the triplet Higgs

$$5^{bar} = \underset{H_T}{(3,1)} + \underset{H_D}{(1,2)}$$



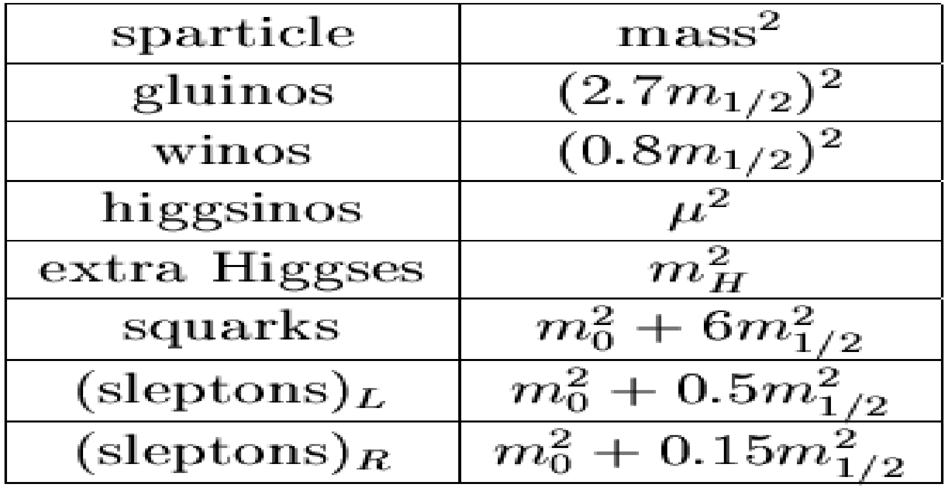
SUSY
threshold

GUT
threshold

Corrections due
to spread of
SUSY multiplets

Corrections due
to spread of
GUT multiplets

From a representative SUSY spectrum:



sparticle	mass^2
gluinos	$(2.7m_{1/2})^2$
winos	$(0.8m_{1/2})^2$
higgsinos	μ^2
extra Higgses	m_H^2
squarks	$m_0^2 + 6m_{1/2}^2$
(sleptons)$_L$	$m_0^2 + 0.5m_{1/2}^2$
(sleptons)$_R$	$m_0^2 + 0.15m_{1/2}^2$

with

$0.8m_0 = 0.8m_{1/2} = 2\mu = m_H = m_{SUSY}$

one finds: $k_{SUSY} \sim -0.510$

The value of k_{GUT} turns out to be negligible for the minimal model $(24+5+5^{bar})$: $k_{GUT} \sim 0$

$k = -0.733 - 0.510 = -1.243$ Minimal Model

This negative k tends to make α_3 too large: we must take m_{SUSY} large and m_T small.

But beware of hierarchy problem and p-decay!

$m_{SUSY} \sim 1$ TeV, $m_T \sim (m_{GUT})^{LO} \longrightarrow$ $\boxed{\alpha_3 \sim 0.13}$ a bit large!

Similarly: $\boxed{M_{GUT} \sim 2\ 10^{16}\ \text{GeV}}$

⑥

KIT, 6-10 February ’12

Beyond the Standard Model

GUT's 2012

Guido Altarelli

Univ. Roma Tre
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Fermion Masses in SU(5)

$m_{Dirac} = \psi_R^{bar} m \, \psi_L$ +h.c $\Rightarrow \psi^c{}_L \psi_L$ +h.c

u: $10 Y^u 10 \cdot H_{5,45bar,50bar}$

d and e: $5^{bar} Y^d 10 \cdot H_{5bar,45}$

ν_{Dirac}: $5^{bar} Y^\nu 1 \cdot H_5$

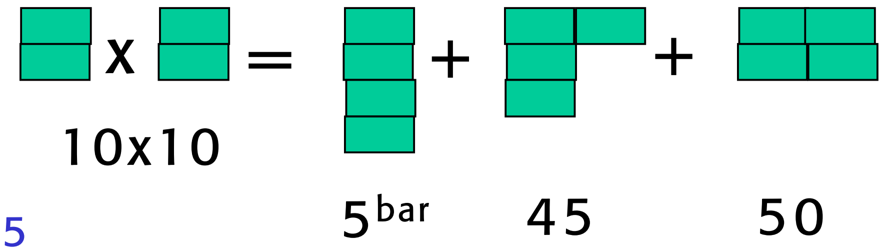
In minimal SU(5) one only has H_5 ($H_{5bar} = H^+$)

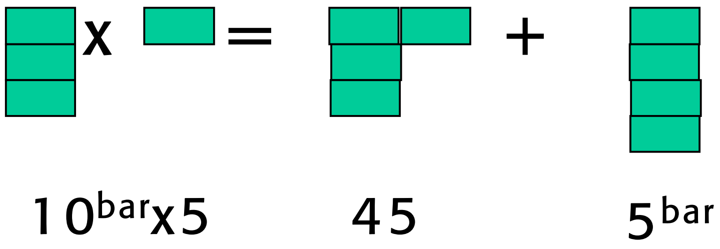
$m_u = Y_u \langle H_5 \rangle$: symmetric

$m_d = m_e^T = Y_d \langle H_5 \rangle$

$10 \times 10 = 5^{bar} + 45 + 50$

$10^{bar} \times 5 = 5^{bar} + 45$





$5^{bar} Y^d 10 \longrightarrow (d^{bar}, L)\ (Q, u^{bar}, e^{bar}) \longrightarrow d^{bar} Q + L e^{bar} + \ldots$

$Q = \begin{bmatrix} u \\ d \end{bmatrix}_L \quad L = \begin{bmatrix} \nu \\ e^- \end{bmatrix}_L$

At M_{GUT}

$m_b / m_\tau = m_s / m_\mu = m_d / m_e$

good bad

bottom-tau unification →

$$\psi_L = \frac{1-\gamma_5}{2}\psi \qquad \overline{\psi_L} = \psi^{\dagger}\frac{1-\gamma_5}{2}\gamma_0 = \bar{\psi}\frac{1+\gamma_5}{2}$$

$$\psi_R = \frac{1+\gamma_5}{2}\psi \qquad \overline{\psi_R} = \psi^{\dagger}\frac{1+\gamma_5}{2}\gamma_0 = \bar{\psi}\frac{1-\gamma_5}{2}$$

$$\psi^c = C\bar{\psi}^T \qquad C = i\gamma_2\gamma_0$$

$$\left(\psi^c\right)_L = \frac{1-\gamma_5}{2}\psi^c = \frac{1-\gamma_5}{2}C\bar{\psi}^T = C\frac{1-\gamma_5}{2}\bar{\psi}^T = C\left(\bar{\psi}\frac{1-\gamma_5}{2}\right)^T = C\overline{\psi_R}^T$$

$$\overline{\psi_R} = \left(\psi^c\right)_L^T C^{-1T} = \left(\psi^c\right)_L^T C$$

$$\overline{\psi_R}\psi_L = \left(\psi^c\right)_L^T C\psi_L$$

for simplicity: $\overline{\psi_R}\psi_L \Rightarrow \psi^c{}_L\psi_L$

Content of SU(5) representations (apart from phases)

$$\bar{5} = \begin{bmatrix} \bar{d}_1 \\ \bar{d}_2 \\ \bar{d}_3 \\ \nu \\ e^- \end{bmatrix} \qquad 10 = \begin{bmatrix} 0 & \bar{u}_3 & \bar{u}_2 & u_1 & d_1 \\ - & 0 & \bar{u}_1 & u_2 & d_2 \\ - & - & 0 & u_3 & d_3 \\ - & - & - & 0 & e^+ \\ - & - & - & - & 0 \end{bmatrix}$$

$$24 = \begin{bmatrix} g & g & g & X_1^{4/3} & Y_1^{1/3} \\ g & g & g & X_2^{4/3} & Y_2^{1/3} \\ g & g & g & X_3^{4/3} & Y_3^{1/3} \\ X_1^{-4/3} & X_2^{-4/3} & X_3^{-4/3} & W^3 & W^+ \\ Y_1^{-1/3} & Y_2^{-1/3} & Y_3^{-1/3} & W^- & B \end{bmatrix}$$

Running masses in SM

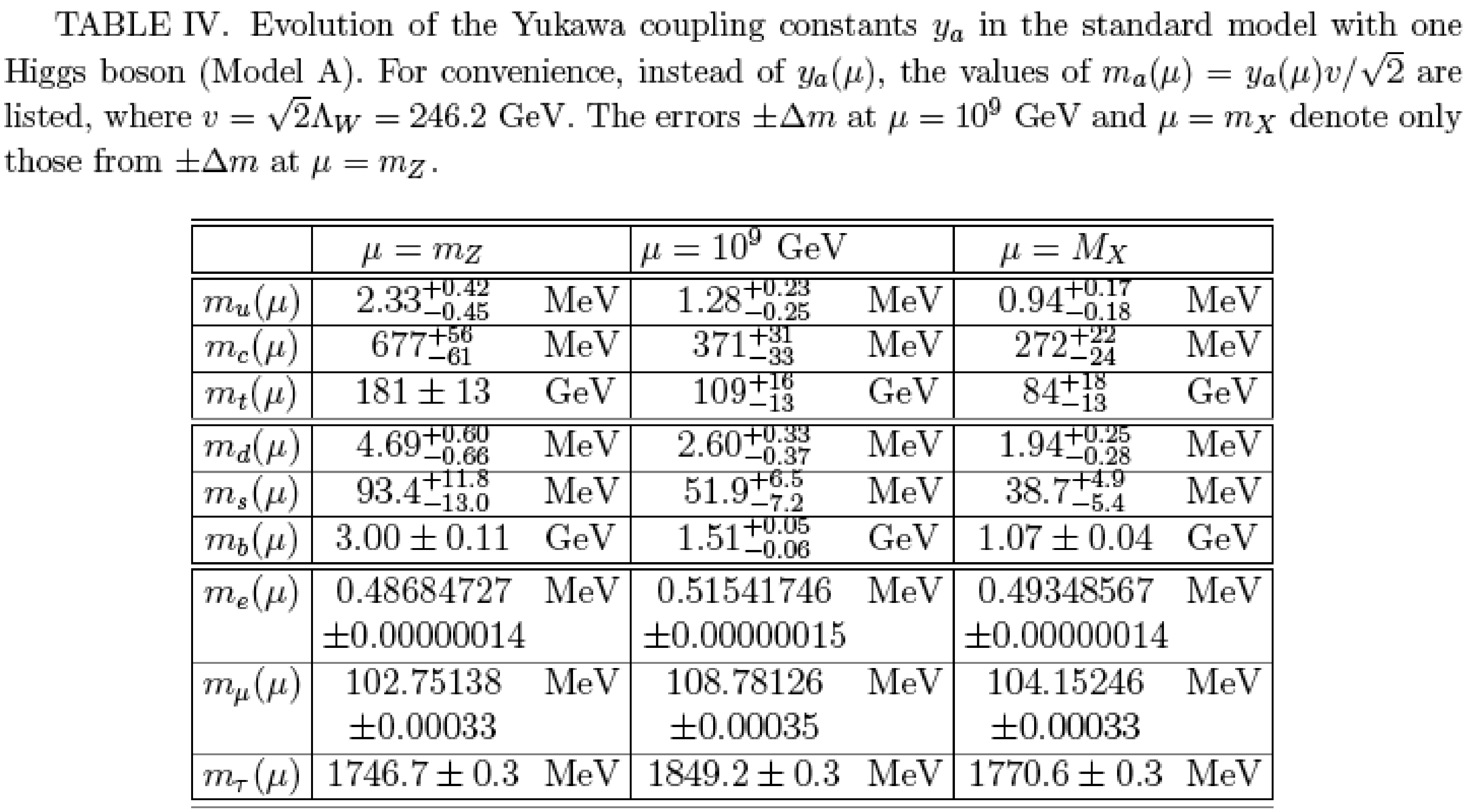


TABLE IV. Evolution of the Yukawa coupling constants y_a in the standard model with one Higgs boson (Model A). For convenience, instead of $y_a(\mu)$, the values of $m_a(\mu) = y_a(\mu)v/\sqrt{2}$ are listed, where $v = \sqrt{2}\Lambda_W = 246.2$ GeV. The errors $\pm\Delta m$ at $\mu = 10^9$ GeV and $\mu = m_X$ denote only those from $\pm\Delta m$ at $\mu = m_Z$.

	$\mu = m_Z$		$\mu = 10^9$ GeV		$\mu = M_X$	
$m_u(\mu)$	$2.33^{+0.42}_{-0.45}$	MeV	$1.28^{+0.23}_{-0.25}$	MeV	$0.94^{+0.17}_{-0.18}$	MeV
$m_c(\mu)$	677^{+56}_{-61}	MeV	371^{+31}_{-33}	MeV	272^{+22}_{-24}	MeV
$m_t(\mu)$	181 ± 13	GeV	109^{+16}_{-13}	GeV	84^{+18}_{-13}	GeV
$m_d(\mu)$	$4.69^{+0.60}_{-0.66}$	MeV	$2.60^{+0.33}_{-0.37}$	MeV	$1.94^{+0.25}_{-0.28}$	MeV
$m_s(\mu)$	$93.4^{+11.8}_{-13.0}$	MeV	$51.9^{+6.5}_{-7.2}$	MeV	$38.7^{+4.9}_{-5.4}$	MeV
$m_b(\mu)$	3.00 ± 0.11	GeV	$1.51^{+0.05}_{-0.06}$	GeV	1.07 ± 0.04	GeV
$m_e(\mu)$	0.48684727 ±0.00000014	MeV	0.51541746 ±0.00000015	MeV	0.49348567 ±0.00000014	MeV
$m_\mu(\mu)$	102.75138 ±0.00033	MeV	108.78126 ±0.00035	MeV	104.15246 ±0.00033	MeV
$m_\tau(\mu)$	1746.7 ± 0.3	MeV	1849.2 ± 0.3	MeV	1770.6 ± 0.3	MeV

Fusuoka, Koide'97

Running masses in MSSM

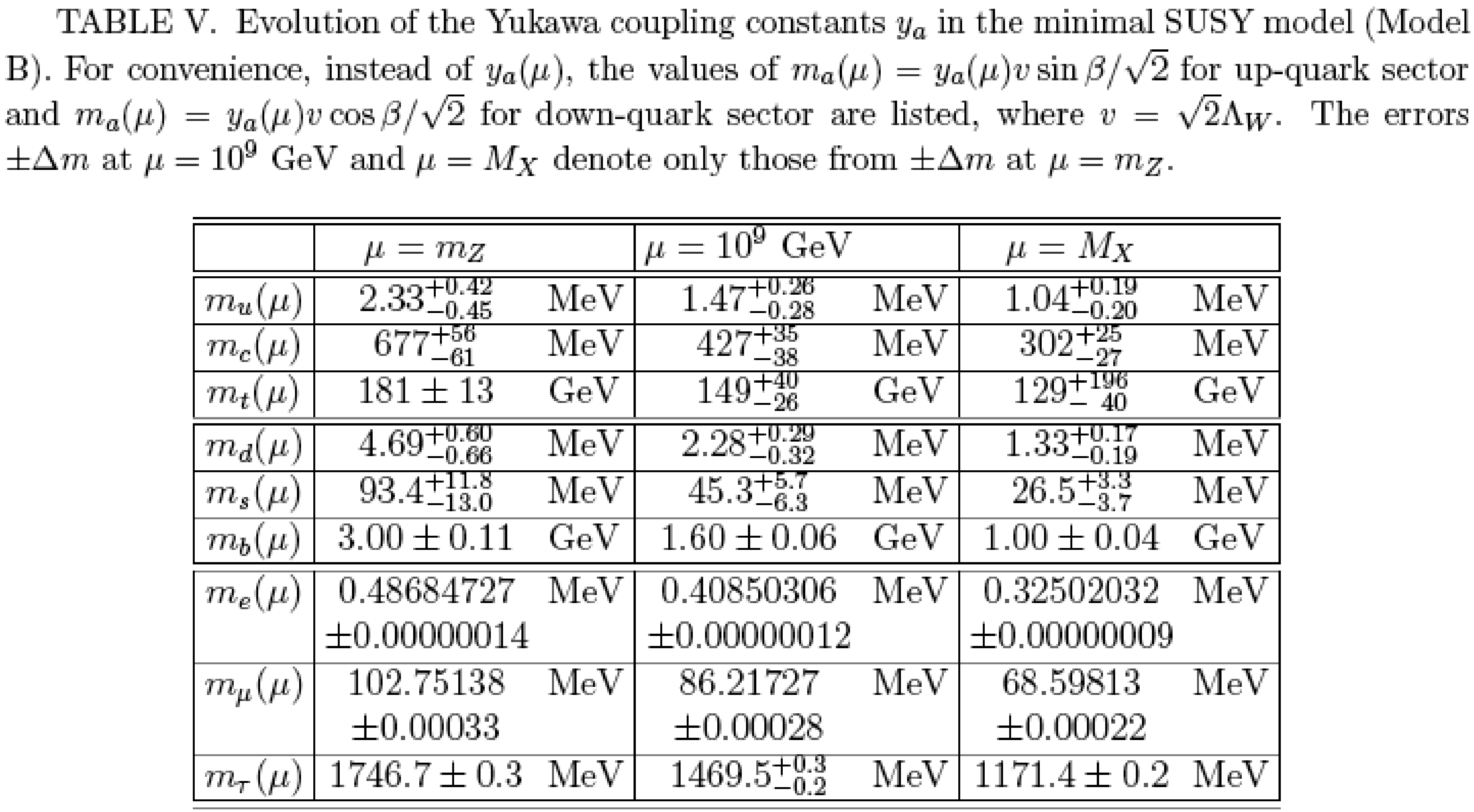
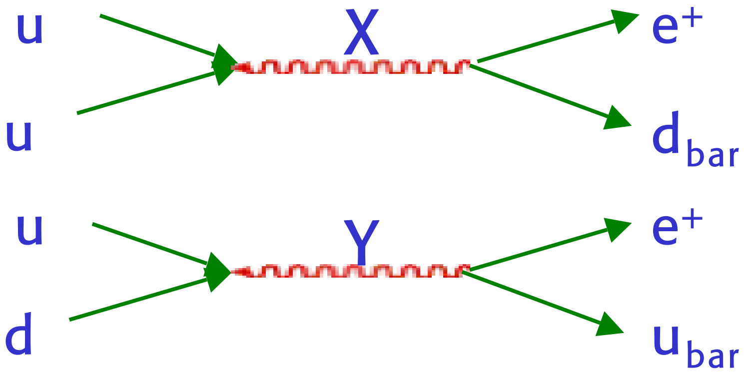


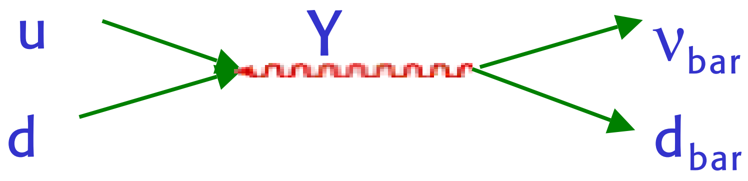
TABLE V. Evolution of the Yukawa coupling constants y_a in the minimal SUSY model (Model B). For convenience, instead of $y_a(\mu)$, the values of $m_a(\mu) = y_a(\mu) v \sin\beta/\sqrt{2}$ for up-quark sector and $m_a(\mu) = y_a(\mu) v \cos\beta/\sqrt{2}$ for down-quark sector are listed, where $v = \sqrt{2}\Lambda_W$. The errors $\pm\Delta m$ at $\mu = 10^9$ GeV and $\mu = M_X$ denote only those from $\pm\Delta m$ at $\mu = m_Z$.

	$\mu = m_Z$	$\mu = 10^9$ GeV	$\mu = M_X$
$m_u(\mu)$	$2.33^{+0.42}_{-0.45}$ MeV	$1.47^{+0.26}_{-0.28}$ MeV	$1.04^{+0.19}_{-0.20}$ MeV
$m_c(\mu)$	677^{+56}_{-61} MeV	427^{+35}_{-38} MeV	302^{+25}_{-27} MeV
$m_t(\mu)$	181 ± 13 GeV	149^{+40}_{-26} GeV	$129^{+196}_{-\ 40}$ GeV
$m_d(\mu)$	$4.69^{+0.60}_{-0.66}$ MeV	$2.28^{+0.29}_{-0.32}$ MeV	$1.33^{+0.17}_{-0.19}$ MeV
$m_s(\mu)$	$93.4^{+11.8}_{-13.0}$ MeV	$45.3^{+5.7}_{-6.3}$ MeV	$26.5^{+3.3}_{-3.7}$ MeV
$m_b(\mu)$	3.00 ± 0.11 GeV	1.60 ± 0.06 GeV	1.00 ± 0.04 GeV
$m_e(\mu)$	0.48684727 MeV ± 0.00000014	0.40850306 MeV ± 0.00000012	0.32502032 MeV ± 0.00000009
$m_\mu(\mu)$	102.75138 MeV ± 0.00033	86.21727 MeV ± 0.00028	68.59813 MeV ± 0.00022
$m_\tau(\mu)$	1746.7 ± 0.3 MeV	$1469.5^{+0.3}_{-0.2}$ MeV	1171.4 ± 0.2 MeV

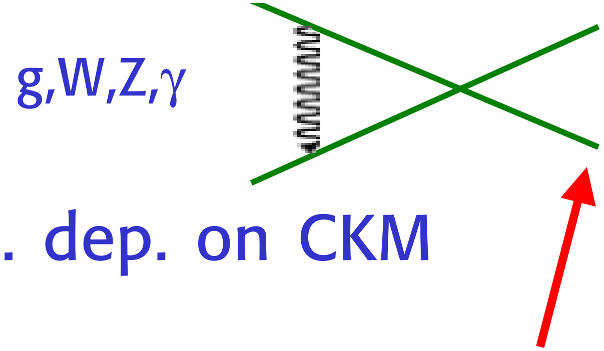
Fusuoka, Koide'97

Proton Decay in SU(5) (no SUSY)





$$\tau_p^{-1} = \Gamma_p \sim \alpha_G^2 \cdot \frac{m_p^5}{M_{X,Y}^4}$$



$p \to e^+\pi^0, e^+\omega, e^+\rho\ldots, \nu^e\pi^+,\ldots$

- Compute the effective 4-f interaction (e.g. dep. on CKM mixing angles)
- Run the vertices from M_{GUT} down to m_p
- Determine $M_{X,Y}$ precisely
- Compute the hadronic matrix element of the 4-f operator (model dep.)

prediction: $\tau_p \sim 10^{30\pm1.7}$ y

exp (SK'11) $p \to e^+\pi^0$:
$\tau_p/B > 1.3\ 10^{34}$y

Non-SUSY SU(5) dead!

Proton Decay in Minimal SUSY-SU(5)

M_{GUT} increases: non SUSY: M_{GUT} $\sim 10^{15}$ GeV, SUSY $\sim 10^{16}$ GeV
and gauge mediation becomes negligible:

$\tau_{p\ \mathrm{NON\ SUSY}} \sim 10^{30\pm1.7}$ y $< 10^{32}$ y

$\tau_{p\ \mathrm{SUSY,\ Gauge}} \sim 10^{36}$ y $(\tau_p \sim m_{GUT}{}^4)$

exp (SK'11) $p \to e^+\pi^0$:
$\tau_p/B > 1.3\ 10^{34}$y

In SUSY coloured Higgs(ino) exchange dominant

Yukawa
Superpot.

$H_{u,d}$: 5 or 5^{bar} H
$G_{u,d}$: matrices in family space

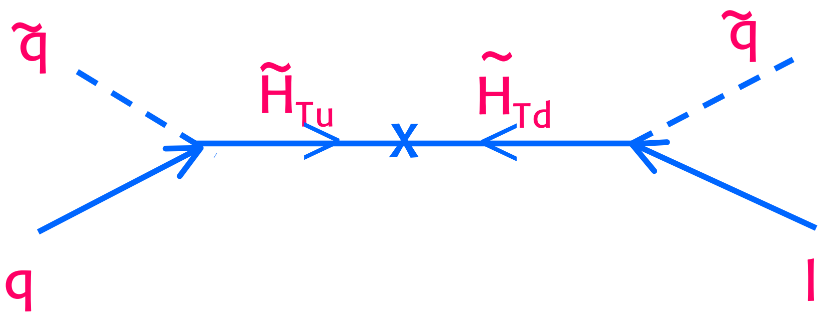
$$W_Y = 1/2\ 10 G_u 10 \cdot H_u + 10 G_d 5 \cdot H^{bar}{}_d$$

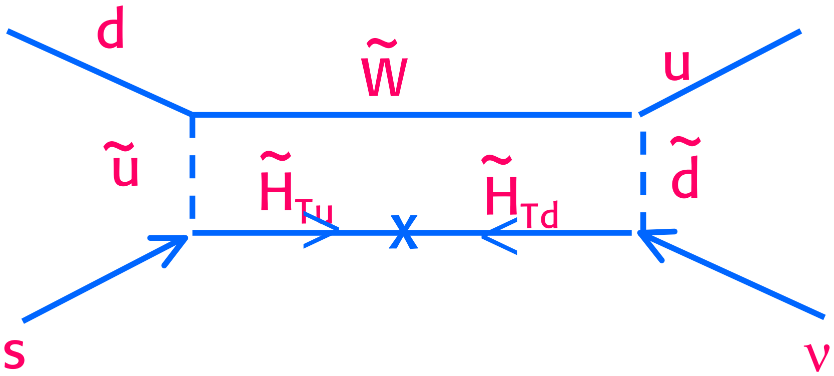
in terms of $H_{D,T}$ (doublet or triplet H):

$$\begin{aligned} W_Y = {} & Q G_u u^c H_{Du} + Q G_d d^c H_{Dd} + e^c G_d{}^T L H_{Dd} + \\ & -1/2\ Q G_u Q H_{Tu} + u^c G_u e^c H_{Tu} - Q G_d L H_{Td} + u^c G_d d^c H_{Td} \end{aligned}$$

The H_D terms -> masses; H_T terms->p-decay

Very rigid:
given the mass constraints p-decay is essentially fixed





After integration of H_T:

Dominant mode $p \to K^+ \nu^{bar}$

$$W_{eff} = [Q(G_u/2)Q \cdot QG_dL + u^cG_ue^c \cdot u^cG_dd^c]/m_{HT}$$

G_u: symm. 3x3 matrix: 12 real parameters

G_d : 3x3 matrix: 18 real parameters

12+18=30 but we can eliminate 9+9 by separately rotating 10 and 5^{bar} fields

3up +3down or lepton masses ($m_l = m_d^T$ in min. SU(5)) + 3 angles+ 1 phase (V_{CKM}) = 10 real parameters

2 phases are the only left-over freedom (arbitrary phases in the 2 terms of W_{eff})

NOT ENOUGH!

In Minimal SUSY-SU(5), using W_{eff} one finds

p-> $K^+\nu^{bar}$ $\tau/B \sim 9\ 10^{32}$ y (exp. > 3 10^{33} y at 90%)

Superkamiokande

This is a central value with a spread of about a factor of about 1/3 - 3.

The minimal model perhaps is not yet completely excluded but the limit is certainly quite constraining.

p decay is a generic prediction of GUT's

establishing B and L non conservation is crucial

Experimental bounds pose severe constraints
Minimal versions are in big trouble:
Minimal non-SUSY is excluded
Minimal SUSY very marginal

$$^{\tau}\!/_{B}(p \rightarrow e^{+} + \pi^{0})_{\exp} > 1.3 \bullet 10^{34}\, yrs$$

$$^{\tau}\!/_{B}(p \rightarrow \overline{\nu} + K^{+})_{\exp} > 3 \bullet 10^{33}\, yrs$$

→ the SUSY mode

One needs either supersymmetry or a GUT-breaking in 2 steps or to introduce specific dynamical ingredients that prevent or suppress p decay

An alternative to SUSY GUT's is 2-scale breaking

We start from a rank-5 group, eg SO(10) and do 2 steps:

$SO(10) \dashrightarrow SU(4)_{PS} x SU(2)_L x SU(2)_R$ at M_{GUT}

and then

$SU(4)_{PS} x SU(2)_L x SU(2)_R \dashrightarrow SU(3)\, x SU(2)_L x U(1)$ at M_I

One typically finds (2-loops, threshold corr's included):

Mohapatra, Parida'93

M_{GUT} moves up to $\sim 10^{16}$ GeV (p decay can be OK)
$M_I \sim 10^{12}$ GeV

(with large uncertainties from thresholds, due to large Higgs representations)

A "realistic" SUSY-GUT model should possess the properties:

- Coupling Unification
  - * No extra light Higgs doublets
  - * M_{GUT} threshold corrections in the right direction
- Natural doublet-triplet splitting
  - * e.g. missing partner mechanism or Dimopoulos-Wilczek
- Well compatible with p-decay bounds
  - * No large fine-tuning
- Correct masses and mixings for q,l and ν's
  - * e.g. $m_b = m_\tau$ at m_{GUT} but m_s different than m_μ, m_d different than m_e

Examples SU(5): Berezhiani, Tavartkiladze; GA, Feruglio, Masina,
GA, Feruglio, Hagedorn......
SO(10): Dermisek, Rabi; Albright, Barr; Ji, Li, Mohapatra;.....

An example of "realistic" SUSY-SU(5)xU(1)$_F$ model

(GA, Feruglio, Masina JHEP11(2000)040; hep-ph/0007254)

The D-T splitting problem is solved by the missing partner mechanism protected from rad. corr's by a flavour symm. U(1)$_F$

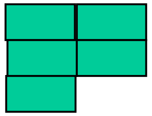
Masiero,Tamvakis; Nanopoulos, Yanagida...

1) We do not want neither the 5.5^{bar} nor the $5.5^{bar}.24$ terms

So, first, we break SU(5) by a 75:

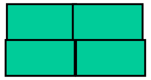
1=X, 75=Y, 5,50=H $_{5,50}$

$$SU(5) \xrightarrow[M_{GUT}]{75} SU(3)x\ SU(2)xU(1)$$


75

2) The 5 5^{bar} Higgs mass term is forbidden by symmetry and masses arise from

$$W=M75.75+75.75.75+5.75.50\ +5^{bar}.75.50^{bar}+50.50^{bar}.1$$


50

As $50=(8,2)+(6,3)+(6^{bar},1)+(3,2)+(3^{bar},1)+(1,1)$

there is a colour triplet (with right charge) but not a colourless doublet (1,2)

→ the doublet finds no partner and only the triplet gets a large mass

Note: we need a large mass for 50 not to spoil coupling unification. But if the terms $5.75.50+ 5^{bar}.75.50^{bar}+50.50^{bar}$ are allowed then also the non rin. operator

$$O = c\,\frac{5\cdot\bar{5}\cdot 75\cdot 75}{M_{Pl}}$$

Randall, Csaki

is allowed in the superpotential and gives too large a mass $M_{GUT}^2/M_{Pl}\sim 10^{12}\text{-}10^{13}$GeV

All this is avoided by taking the following $U(1)_F$ charges :

Berezhiani, Tavartkiladze

field:	Y_{75}	H_5	H_{5bar}	H_{50}	H_{50bar}	X_1
F-ch:	0	-2	1	2	-1	-1

All good terms are then allowed:

$W=M75.75+75.75.75+5.75.50+ 5^{bar}.75.50^{bar}+50.50^{bar}.1$

while all bad terms like $5.5^{bar}.(X)^n.(Y)^m$, n,m>0 are forbidden

Coupling unification

Recall:

1-loop

$$\alpha_3 = \frac{\alpha_3^{LO}}{1+\alpha_3^{LO}\delta}$$

$$\delta = k + \frac{1}{2\pi}\log\frac{m_{SUSY}}{m_Z} - \frac{3}{5\pi}\log\frac{m_{H_T}}{m_{GUT}^{LO}}$$

$$k = k_2 + \underbrace{k_{SUSY} + k_{GUT}}_{\text{thresholds}}$$

2-loop

k_2 ~ -0.733, k_{SUSY} ~ -0.510 remain the same.
But k_{GUT} ~ 0 for the 24 is now k_{GUT} ~ 1.86 for the 75 (the 50 is unsplit).
So k ~ -1.243 in the minimal model becomes k~+0.614 in this model.

Now α_s would become too small and we need m_{SUSY} small and m_T large

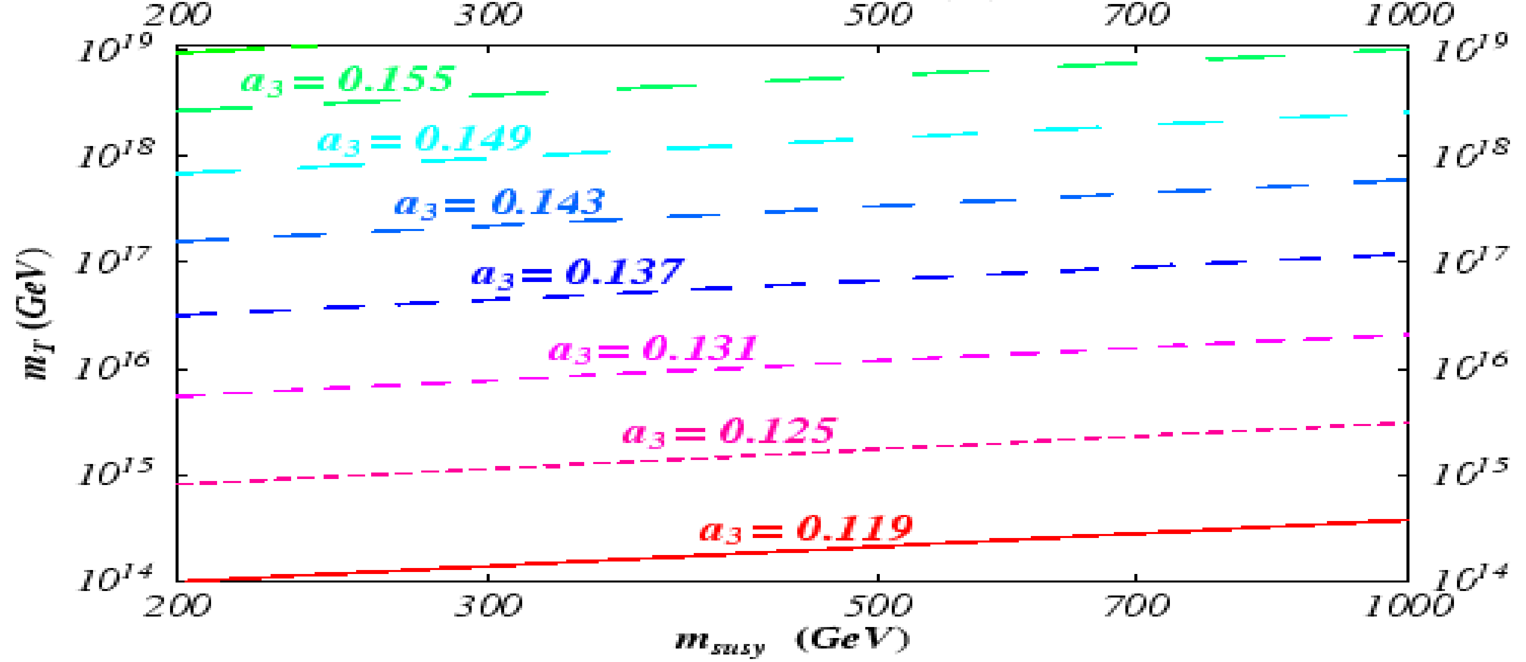
$m_T|_{Realistic}$ ~ 20-30 $m_T|_{minimal}$

good for p-decay!
factor 400-900

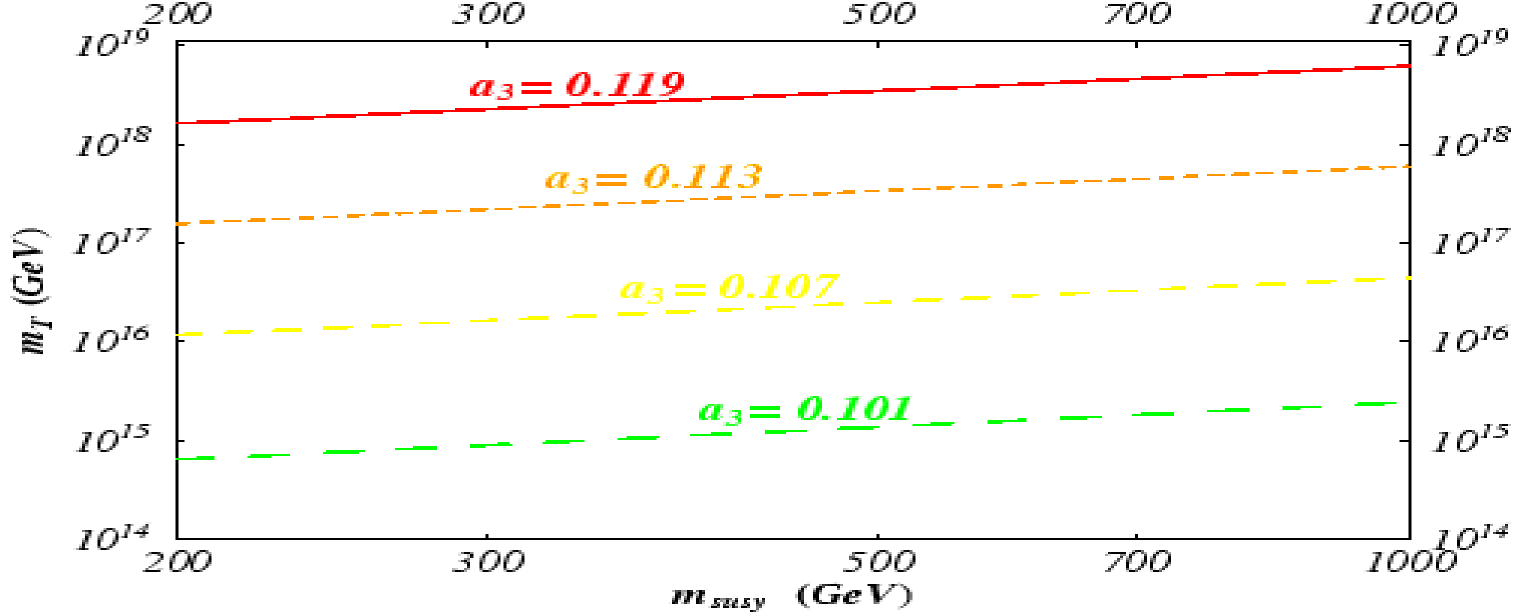
Due to 50, 75, SU(5) no more asympt. free: α_s blows up below m_{Pl} (Λ~20-30 M_{GUT})

Not necessarily bad!

Minimal SU(5)



Missing Doublet



Fermion masses

$F(X,Y)$

Consider a typical mass term: $10G_d5^{bar}H_d$

Recall: X SU(5) singlet, $F(X) = -1$
Y SU(5) 75, $F(Y) = 0$

First approximation:
no Y insertions -> $F(X,0)$

Pattern determined by $U(1)_F$ charges

Froggatt-Nielsen

i,j=family1,2,3

$F(10) = (4,3,1)$ $F(H_u) = -2$
$F(5^{bar}) = (4,2,2)$ $F(H_d) = 1$
$F(1) = (4,-1,0)$

$\lambda_C \sim 0.22$

$$10_i5^{bar}{}_j(<X>/\Lambda)^{fi+fj+fH}v_d$$

$$m_u = \begin{bmatrix} \lambda^6 & \lambda^5 & \lambda^3 \\ \lambda^5 & \lambda^4 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{bmatrix} v_u \qquad m_d = m_l^T = \begin{bmatrix} \lambda^5 & \lambda^3 & \lambda^3 \\ \lambda^4 & \lambda^2 & \lambda^2 \\ \lambda^2 & 1 & 1 \end{bmatrix} v_d\lambda^4$$

quarks: m_u, m_d, V_{CKM} ~OK, $tg\beta \sim o(1)$
ch. leptons: $m_d = m_l^T$ broken by Y insertions

1st order:

$$10_i5^{bar}{}_j\lambda_C{}^{nij}(<Y>/\Lambda)\, v_d$$

$m_d \sim G_d + <Y>/\Lambda F_d$
($m_e^T \sim G_d - 3<Y>/\Lambda F_d$

Hierarchy for masses and mixings via horizontal $U(1)_F$ charges.

Froggatt, Nielsen '79

Principle: A generic mass term

$$\bar{R}_1 m_{12} L_2 H$$

is forbidden by U(1)
if $q_1+q_2+q_H$ not 0

q_1, q_2, q_H:
U(1) charges of
$\bar{R}_1, L_2, H$

U(1) broken by vev of "flavon" field θ with U(1) charge $q_\theta = -1$.
If vev $\theta = w$, and $w/M=\lambda$ we get for a generic interaction:

$$\bar{R}_1 m_{12} L_2 H\, (\theta/M)^{\overset{\Delta_{charge}}{q1+q2+qH}} \qquad m_{12} \to m_{12}\, \lambda^{q1+q2+qH}$$

Hierarchy: More Δ_{charge} -> more suppression (λ small)

One can have more flavons (λ, λ', ...)
with different charges (>0 or <0) etc -> many versions

Proton decay

Higgs triplet exchange

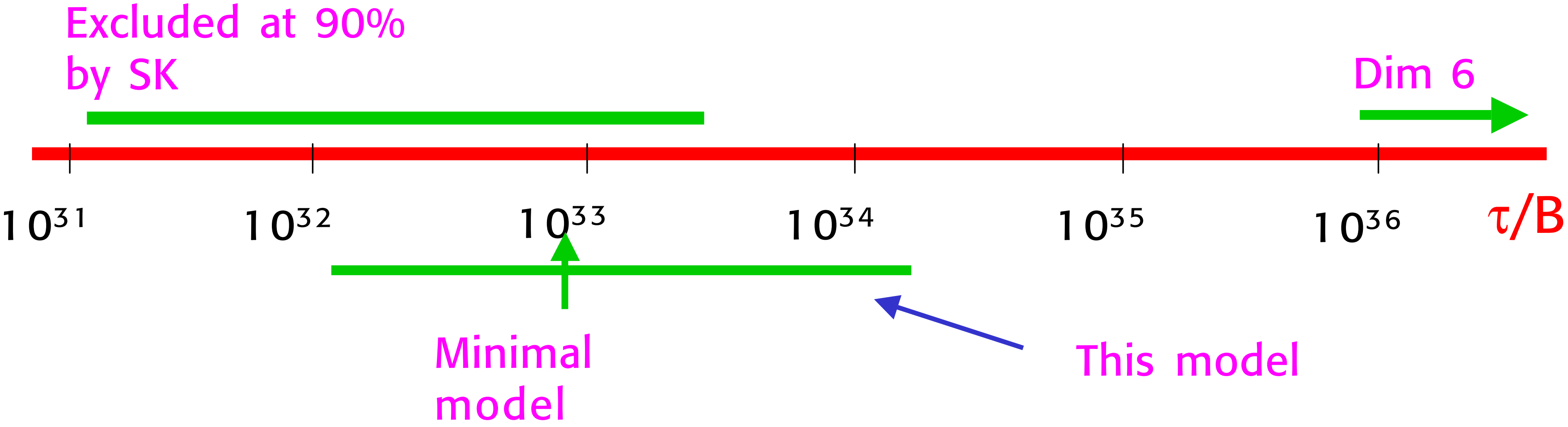
$W_{eff} = [Q(1/2A)Q\cdot QBL + u^cCe^c\cdot u^cDd^c]/m_{HT}$

Advantages w.r.t. minimal SUSY-SU(5)

- Larger m_T by factor 20 -30
- Extra terms: e.g. not only $10G_u 10H_u$ but also $10G_{50}10H_{50bar}$

(free of mass constraints because $<H_{50bar}>=0$)

Results: $p \to K^+\nu_{bar}$ (similarly for $p \to \pi^0 e^+$)



Excluded at 90% by SK

Dim 6

Minimal model

This model

Mass terms in SO(10)

16x16 = 10+126+120

H Δ Σ

120 is antisymm

Renormalisable mass terms

$$W_Y = h\,\psi\psi H + f\,\psi\psi\bar{\Delta} + h'\,\psi\psi\Sigma,$$

h, f symm. matrices, h' antisymm.

H, Δ and Σ contain 2, 2 and 4 Higgs doublets, resp.

Only 1 H_u and 1 H_d remain nearly massless

$$\begin{aligned} Y_u &= h + r_2 f + r_3 h', \\ Y_d &= r_1 (h + f + h'), \\ Y_e &= r_1 (h - 3f + c_e h'), \\ Y_{\nu D} &= h - 3 r_2 f + c_\nu h', \end{aligned}$$

Minimal SO(10) (only H) predicts

$m_u = m_{\nu D}$ too restrictive

$m_d = m_e$

To avoid large Higgs representations higher dimension non renormalizable couplings can be used

As 10x45 =10+120+320 and 16x16=10+120+126

$$W_Y = h\,\psi\psi H + f\,\psi\psi\underset{\substack{\uparrow\\ H_{16}xH_{16}}}{\bar{\Delta}} + h'\,\psi\psi\underset{\substack{\nwarrow\\ H_{10}xH_{45}\\ H_{16}xH_{16}}}{\Sigma},$$

In this case f and h' are suppressed by 1/M

Dimopoulos-Wilczek mechanism for doublet triplet splitting in SO(10)

Introduce a 45 with vev

$$\langle 45 \rangle = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \otimes Diag(M, M, M, 0, 0)$$

with M ~ 0(M_{GUT}), in basis where

$$10 = \begin{pmatrix} 5 \\ \bar{5} \end{pmatrix} = \begin{pmatrix} H_T \\ H_D \\ K_T \\ K_D \end{pmatrix}$$

We need two ten's 10, 10'
because 45 is antisymm.

10 45 10' gives a large mass to the triplets and not to the doublets

Then one must raise the mass of two of the doublets

- Minimal SUSY-SU(5), -SO(10) models are in trouble
- More realistic models are possible but they tend to be baroque (e.g. large Higgs representations)

Recently a new idea has been developed and looks promising:

unification in extra dimensions

[Fayet '84],
Kawamura '00
GA, Feruglio '01
Hall, Nomura '01
Hebecker, March-Russell '01;
Hall, March-Russell, Okui, Smith
Asaka, Buchmuller, Covi '01
....

R: compactification radius

compact

Factorised metric

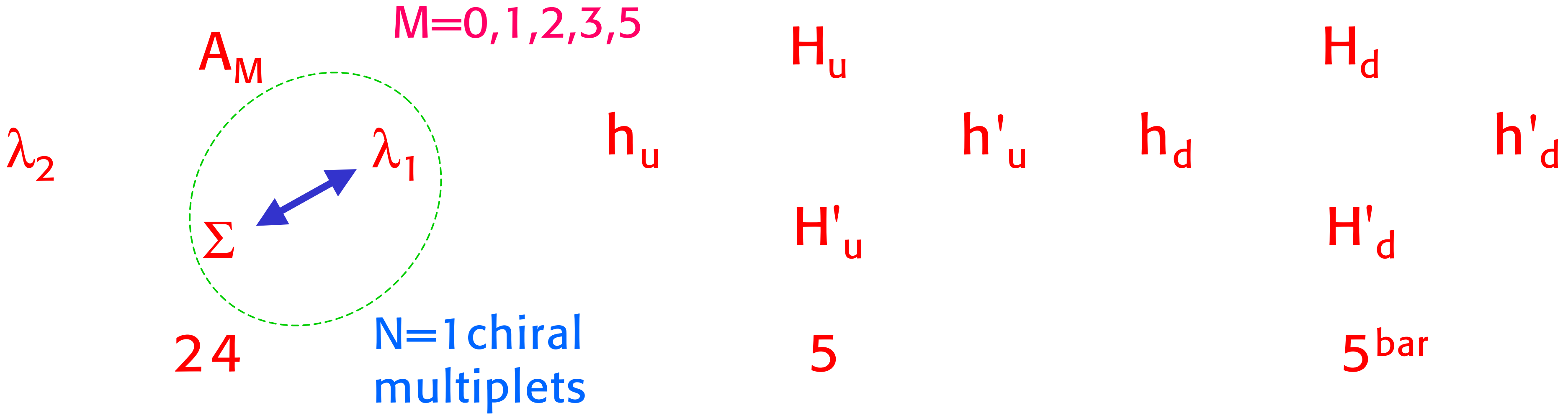
$$ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu + h_{ij}(y) dy^i dy^j$$

But while for the hierarchy problem R is much larger here we consider R ~ $1/M_{GUT}$ (not so large!)

A different view of GUT's SUSY-SU(5) in extra dimensions

- In 5 dim. the theory is symmetric under N=2 SUSY and SU(5)

Gauge 24 + Higgs $5+5^{bar}$: N=2 supermultiplets in the bulk



M=0,1,2,3,5

A_M

λ_2 Σ λ_1

24

N=1 chiral multiplets

H_u h_u h'_u H'_u

5

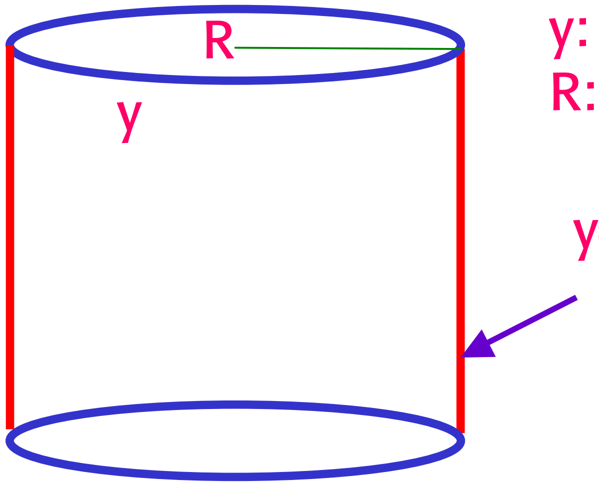
H_d h_d h'_d H'_d

5^{bar}

- Compactification by $S/(Z_2xZ_2')$ $1/R \sim M_{GUT}$

N=2 SUSY-SU(5) -> N=1 SUSY-SU(3)xSU(2)xU(1)

- Matter 10, 5^{bar}, 1 on the brane (e.g. $x_5=y=0$) or in the bulk (many possible variations)



y: extra dimension
R: compact'n radius

y=0 "our" brane

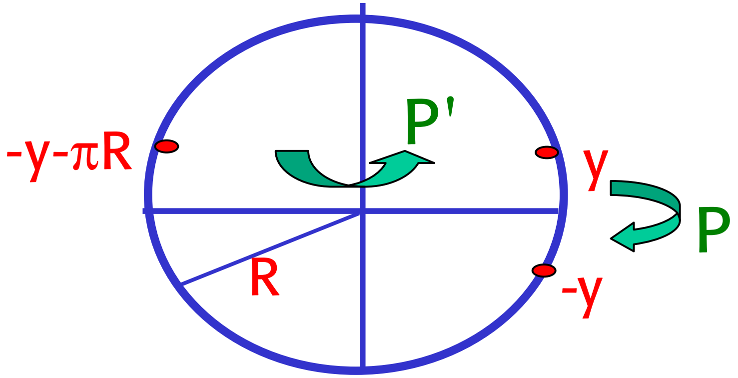
$S/(Z_2 x Z_2')$

Z_2-> P: y ⟷ -y

Z_2'-> P': y' ⟷ -y'

y' = y + πR/2

or y ⟷ -y - πR



Diagonal fields in P,P' can be Fourier expanded:

$$\phi_{++}(x_\mu, y) = \sqrt{\frac{2}{\pi R}} \cdot \sum_n \phi_{++}^{(2n)}(x_\mu) \cos\frac{2ny}{R}$$

$$\phi_{+-}(x_\mu, y) = \sqrt{\frac{2}{\pi R}} \cdot \sum_n \phi_{+-}^{(2n+1)}(x_\mu) \cos\frac{2n+1}{R}y$$

$$\phi_{-+}(x_\mu, y) = \sqrt{\frac{2}{\pi R}} \cdot \sum \phi_{-+}^{(2n+1)}(x_\mu) \sin\frac{2n+1}{R}y$$

$$\phi_{--}(x_\mu, y) = \sqrt{\frac{2}{\pi R}} \cdot \sum_n \phi_{--}^{(2n+2)}(x_\mu) \sin\frac{2n+2}{R}y$$

Only ϕ_{++},ϕ_{+-} not 0 at y=0

Only ϕ_{++} is massless

P breaks N=2 SUSY down to N=1 SUSY
but conserves SU(5): on 5 of SU(5) $P=(+,+,+,+,+)$

P' breaks SU(5) $P'=(-,-,-,+,+)$ $P'T^aP'=T^a$, $P'T^\alpha P'= -T^\alpha$
(T^a: span 3x2x1, T^α: all other SU(5) gen.'s)

P P'	bulk field	mass
++	$A^a_\mu, \lambda^a_2, H^D_u, H^D_d$	2n/R
+ -	$A^\alpha_\mu, \lambda^\alpha_2, H^T_u, H^T_d$	(2n+1)/R
- +	$A^\alpha_5, \Sigma^\alpha, \lambda^\alpha_1, H'^T_u, H'^T_d$	(2n+1)/R
- -	$A^a_5, \Sigma^a, \lambda^a_1, H'^D_u, H'^D_d$	(2n+2)/R

← Doublet
← Triplet

Note:
$\partial_5 = (-,-)$

Gauge parameters are also y dep.

$$U = \exp[i\xi^a(x_\mu, y)T^a + i\xi^\alpha(x_\mu, y)T^\alpha]$$

$$\xi^a(x_\mu, y) = \sqrt{\frac{2}{\pi R}} \cdot \sum \xi^a(x_\mu)\cos\frac{2ny}{R}$$

$$\xi^\alpha(x_\mu, y) = \sqrt{\frac{2}{\pi R}} \cdot \sum_n \xi^\alpha(x_\mu)\cos\frac{2n+1}{R}y$$

} both not zero at y=0

$$U = \exp[i\xi^a(x_\mu, y)T^a + i\xi^\alpha(x_\mu, y)T^\alpha]$$

$$\xi^a(x_\mu, y) = \sqrt{\frac{2}{\pi R}} \cdot \sum_n \xi^a(x_\mu)\cos\frac{2ny}{R}$$

$$\xi^\alpha(x_\mu, y) = \sqrt{\frac{2}{\pi R}} \cdot \sum_n \xi^\alpha(x_\mu)\cos\frac{2n+1}{R}y$$

At $y=0$ both ξ^a and ξ^α not 0: so full SU(5) gauge transf.s, while at $y=\pi R/2$ only SU(3)xSU(2)xU(1).

Virtues:

- No baroque 24 Higgs to break SU(5)
- $A^{a(0)}_\mu, \lambda^{a(0)}_2$ massless N=1 multiplet
- $A^{a(2n)}_\mu$ eat $\partial_5 A^{a(2n)}_5$ and become massive (n>0)
- Doublet-Triplet splitting automatic and natural:

$H^{D(0)}_{u,d}$ massless, $H^{T(0)}_{u,d}$ $m \sim 1/R \sim m_{GUT}$

The brane at $y=0$ (or πR) is a fixed point under P.
There the full SU(5) gauge group operates.
The brane at $y= \pi R/2$ (or $- \pi R/2$) is a fixed point under P'. There only the SM gauge group operates.

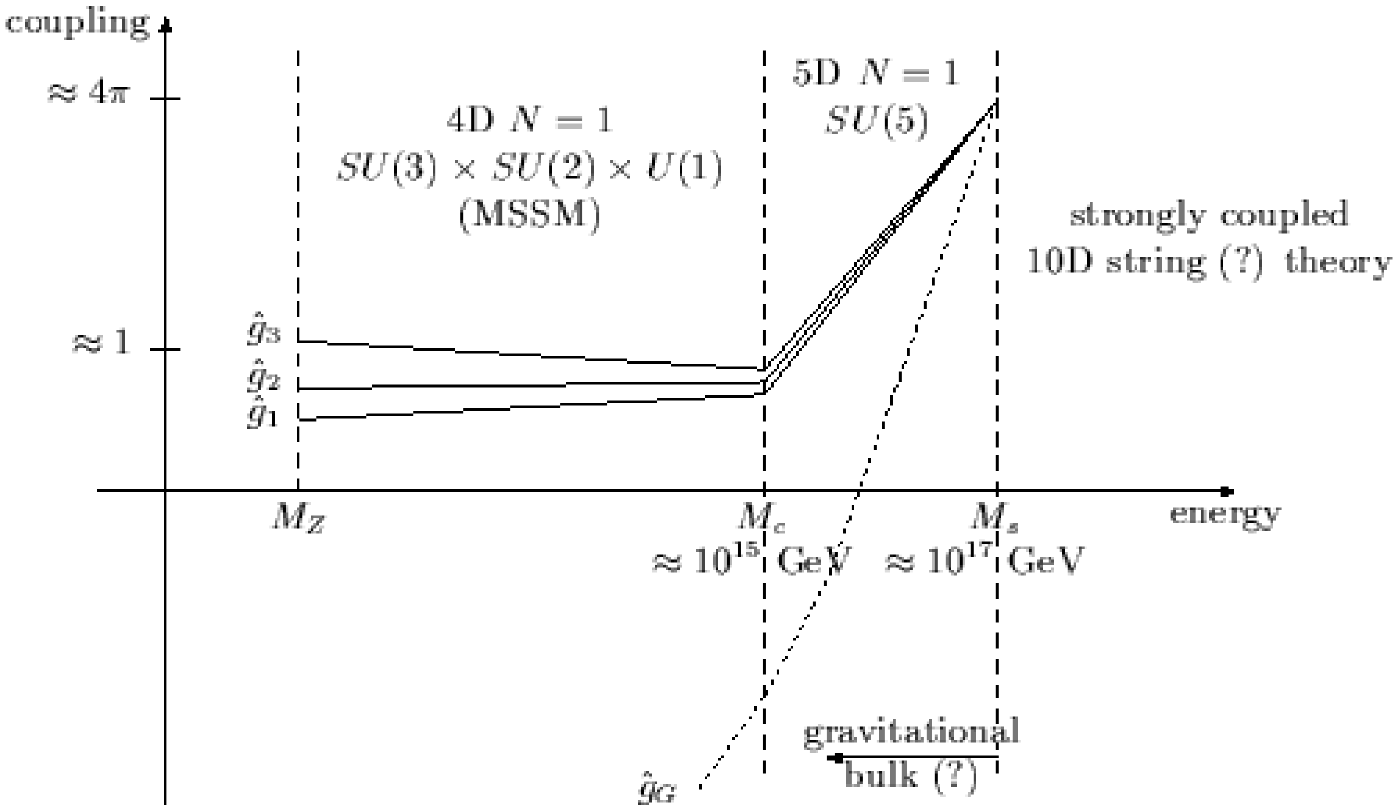
Matter fields (10, 5^{bar}, 1, and the Higgs also) could be either on the bulk, or at $y=0$ or $y= \pi R/2$. Many possibilities

In the bulk must satisfy all symmetries, at $y=0$ must come in N=1 SUSY-SU(5) representations, at $y= \pi R/2$ must only fill N=1 SUSY-SU(3)xSU(2)xU(1) representations

For example, if H^D_u, H^D_d are at $y= \pi R/2$ one can even not introduce H^T_u, H^T_d

Coupling unification can be maintained and threshold corrections evaluated

Hall, Nomura
Contino, Pilo, Rattazzi, Trincherini



SO(10) models can also be constructed

Breaking by orbifolding requires 6-dim and leave an extra U(1)
(the rank is maintained)

Asaka, Buchmuller, Covi
Hall, Nomura

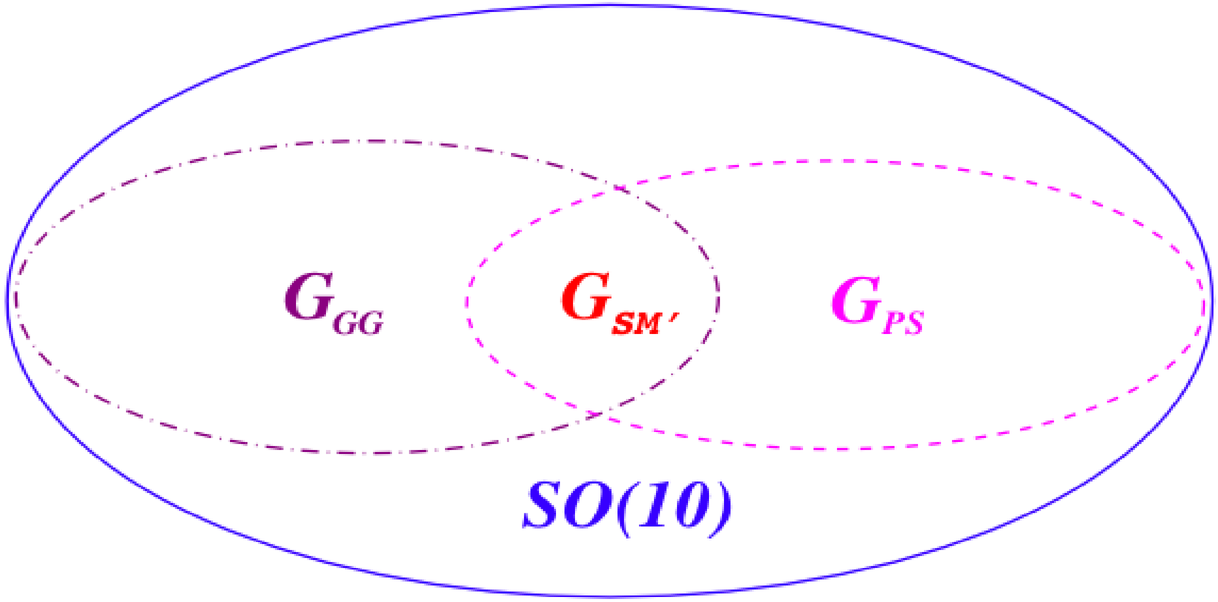
Breaking by BC or mixed orbifolding+BC can be realised in 5 dimensions

Dermisek, Mafi;
Kim, Rabi
Albright, Barr
Barr, Dorsner (flipped SU(5))

Breaking SUSY-SO(10) in 6 dim by orbifolding

The ED y, z span a torus $T^2 \rightarrow T^2/Z \times Z_{PS} \times Z_{GG}$

$$G_{PS} = SU(4) \times SU(2) \times SU(2)\ , \quad G_{GG} = SU(5) \times U(1)_X$$



$$G_{SM'} = SU(3) \times SU(2) \times U(1) \times U(1)$$

Thus:

- By realising GUT's in extra-dim we obtain great advantages:
  - No baroque Higgs system
  - Natural doublet-triplet splitting
  - Coupling unification can be maintained (threshold corr.'s can be controlled)
  - P-decay can be suppressed or even forbidden
  - SU(5) mass relations can be maintained, or removed (also family by family)

Conclusion

Grand Unification is a very attractive idea

Unity of forces, unity of quarks and leptons
explanation of family quantum numbers,
charge quantisation, B&L non conservation
(baryogenesis)

Coupling unification: SUSY [SU(5) or SO(10)] or
2-scale breaking in SO(10) no-SUSY

Minimal models in trouble

Realistic models mostly baroque

GUT's in ED offer an example of a more complex reality

BACKUP

SU(N) representations   First recall SU(3)

$q'_a = U_a{}^b q_b$ In the fund. repr. 3 SU(3) is mapped by the 3x3 matrices U with $U^+U=1$ and det $U=1$

A tensor with n (lower) indices transforms as $q_{a1}q_{a2}\dots q_{an}$:

$$T'_{a1a2\cdots an} = U_{a1}{}^{b1}U_{a2}{}^{b2}\dots\dots U_{an}{}^{bn}T_{b1b2\dots bn}$$

Thus a definite symmetry is maintained in the transf. ---> irreducible tensors have definite symmetry

e.g. $3x3 \dashrightarrow T_{\{ab\}} + T_{[ab]} = 6 + 3^{bar}$

{ } : symm.
[ ] : antisymm.

ε_{abc} is an invariant in SU(3):

$$\varepsilon'_{abc} = U_a{}^{a'}U_b{}^{b'}U_c{}^{c'}\varepsilon_{a'b'c'} = \mathrm{Det}U\ \varepsilon_{abc} = \varepsilon_{abc}$$

So $\varepsilon^{abc}q_a q_b q_c$ is an invariant in SU(3). [3x3x3 contains 1: in QCD colour singlet baryons are $\varepsilon^{abc}q_a q_b q_c$]

(We set $\varepsilon^{abc} = \varepsilon_{abc}$)

We can define higher indices starting from: $q^a = \varepsilon^{abc}q_b q_c$

Then $q^a q_a$ is an invariant. This implies that $q'^a = U^{*a}{}_b q^b$

In fact $q'^a q'_a = U^{*a}{}_b U_a{}^c q^b q_c = q^a q_a$ (because of $U^+U=1$)

So $\delta^a{}_b = \delta_a{}^b$ is an invariant.

In general: $T'^{a1a2...an} = U^{*a1}{}_{b1} U^{*a2}{}_{b2}.....U^{*an}{}_{bn} T^{b1b2....bn}$

The most general irreducible tensor in SU(3) has n symmetric lower and m symmetric higher indices with all traces subtracted (in SU(N>3) antisymm. indices cannot be all eliminated)

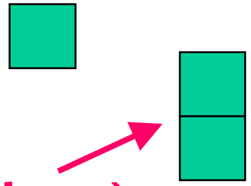
Products of repr.ns and Young Tableaux in SU(3)

$3 \mathrm{x} 3 = 6 + 3^{\mathrm{bar}}$

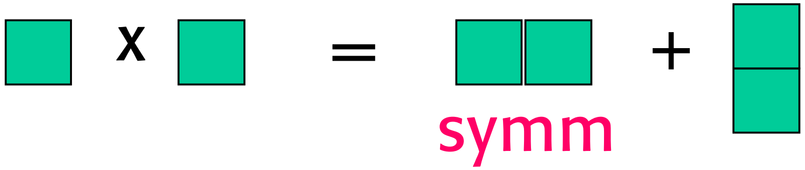
$q_a \mathrm{x} q_b = q_{\{a} q_{b\}} + q_{[a} q_{b]}$

3 ~ a lower index ~

3^{bar} ~ a higher index ~

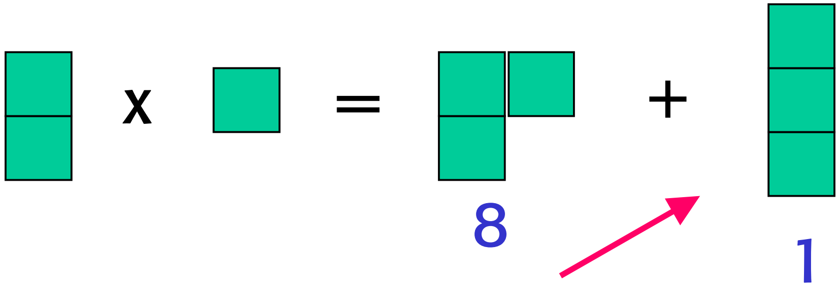


(~ 2 antisymm. indices)



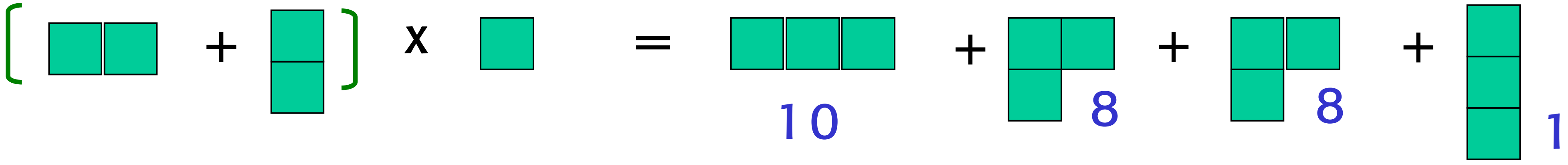
$3 \mathrm{x} 3^{\mathrm{bar}} = 3^{\mathrm{bar}} \mathrm{x} 3 = 8 + 1$

$q^a \mathrm{x} q_b = q^a q_b - 1/3\, \delta^a_b\, q^c q_c$

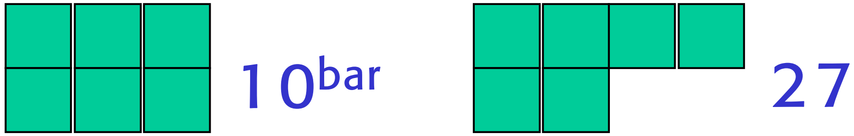


a singlet:
3 antisymm. indices

$3 \mathrm{x} 3 \mathrm{x} 3 = (6 + 3^{\mathrm{bar}}) \mathrm{x} 3 = 10 + 8 + 8 + 1$

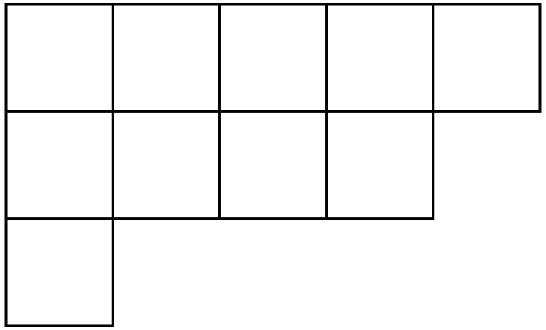


⊕ $8 \mathrm{x} 8 = 1 + 8 + 8 + 10 + 10^{\mathrm{bar}} + 27$



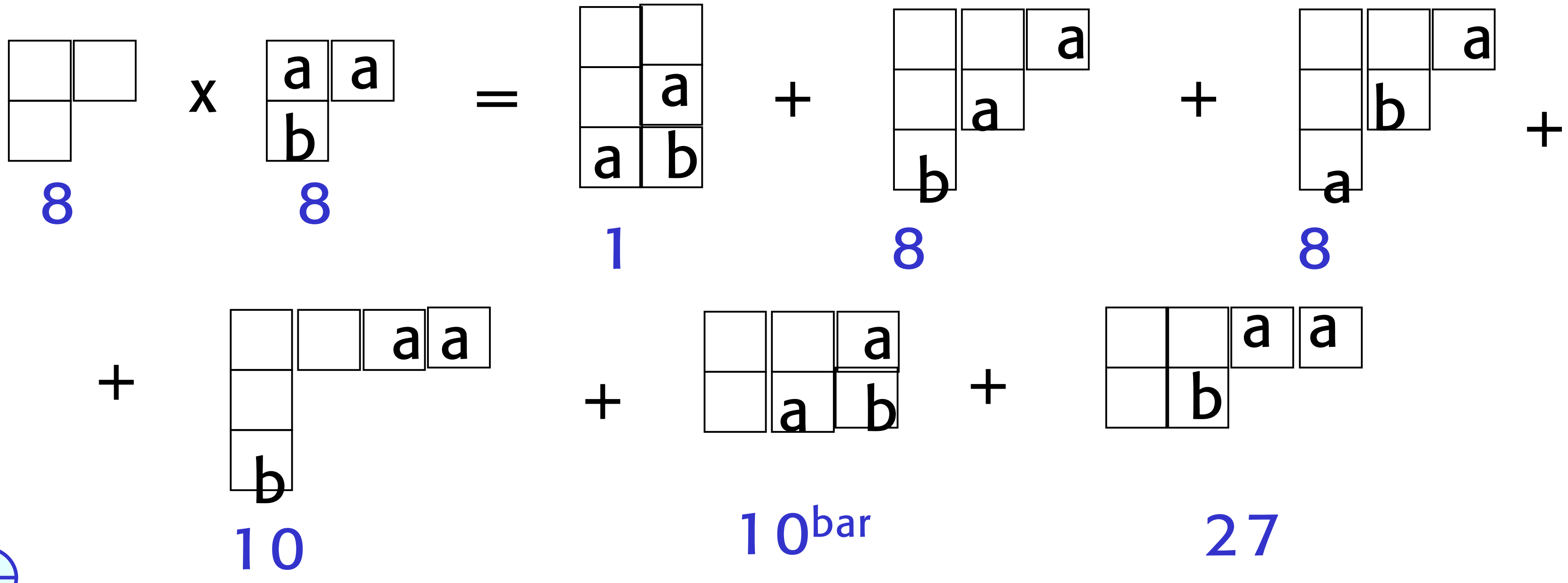
A Young tableau is always of the form:

longer columns ordered from the left



doing products, symmetrized indices (on the same row) should not be placed on a column (that is, antisymmetrized)

Example in SU(3):



In SU(2) 2 and 2^{bar} are equivalent: U and U* are related by a unitary change of basis

$\varepsilon_{ab} = \varepsilon^{ab} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} = \varepsilon$ $\varepsilon\varepsilon^{+} = 1$ $\varepsilon^{+} = -\varepsilon$

$\varepsilon U \varepsilon^{+} = U^{*}$ $U = \exp(i\frac{\vec{\tau}}{2}\vec{\theta})$ τ: Pauli matrices

In fact: $\varepsilon\tau\varepsilon^{+} = -\tau^{*}$

In SU(N) a higher index is equivalent to N-1 lower antisymmetric indices.

$T^{a} = \varepsilon^{a\ b1\ b2.....bN-1} T_{b1\ b2....\ bN-1}$

In SU(5) 3 lower antisymm. indices ~ 2 upper antisymm.

Consider G with rank 4: SU(5), SU(3)x SU(3)

SU(3)x SU(3) cannot work. One SU(3) must be SU(3)$_{colour}$. The weak SU(3) commutes with colour -> q, q^{bar}, and leptons in diff. repr.ns. But TrQ=0, so, for example

q ~(u,d, D), q^{bar} ~(u^{bar},d^{bar}, D^{bar}), l ~ octet

where D is a new heavy Q=-1/3 coloured, isosinglet quark.

But then $Tr(T^3)^2=3/2$, $TrQ^2=2$ and:

$$1+b^2 = \frac{TrQ^2}{TrT_3^2} = \frac{4}{3} \longrightarrow s_W^2\Big|_{at\ M} = \frac{1}{1+b^2} = \frac{3}{4}$$

Too large! (was 3/8)

Also weak W± currents cannot be pure V-A because antiquarks cannot be singlets (TrQ=Q not 0).

Note that SU(3)xSU(3)xSU(3) could work: $Q=T_L+T_R+(Y_L+Y_R)/2$

$(3,3^{bar},1) + (3^{bar},1,3) + (1,3,3^{bar})$

q anti-q leptons

In a parity doublet trQ^2 is twice and trT$_L^2$ is the same: $s_W^2=3/8$

In the SUSY limit <5>, $<5^{bar}>$, <50>, $<50^{bar}>$=0 while $<Y> \sim M_{GUT}$ and <X> is undetermined. Higgs doublets stay massless. Triplet Higgs mix between 5 and 50:

$$m_T = \begin{bmatrix} 0 & 5.50 \\ 5.50 & 50.50 \end{bmatrix} = \begin{bmatrix} 0 & <Y> \\ <Y> & <X> \end{bmatrix}$$

In terms of $m_{T1,2}$ (eigenvalues of $m_T m_T^+$) the relevant mass for p-decay is

$$m_T = \frac{m_{T1} \cdot m_{T2}}{<X>} \sim \frac{<Y>^2}{<X>}$$

When SUSY is broken the doublets get a small mass and <X> is driven at the cut-off between m_{GUT} and m_{Pl}.

A simple option is to take the Higgs in the bulk and the matter 10, 5^{bar}, 1 at $y=0$, πR.

In our paper we take fully symmetric Yukawa couplings at $y=0$:

$$W_Y = 1/2\ 10 G_u 10 \cdot H_u + 10 G_d 5 \cdot H^{\mathrm{bar}}{}_d$$

This contains H^D (mass) and H^T (p-decay) interactions:

$$W_D = Q G_u u^c \cdot H^D{}_u + Q G_d d^c \cdot H^D{}_d + L G_d e^c \cdot H^D{}_d$$

$$W_T = Q G_u Q \cdot H^T{}_u + u^c G_d d^c \cdot H^T{}_d + Q G_d L \cdot H^T{}_d + u^c G_u e^c \cdot H^T{}_u$$

P' transforms $y=0$ into $y=\pi R$. We choose P' parities of 10, 5^{bar}, 1 that fix $W(y=\pi R)$ such that only wanted terms survive in

$$w^{(4)} = \int [\delta(y) + \delta(y - \pi R)] w(y) dy$$

We take Q, u^c, d^c +,+ and L, e^c, ν^c +,-:

recall H^D ++, H^T +-

all mass terms allowed, p-decay forbidden

QQQL, $u^c u^c d^c e^c$, $Q d^c L$, $L e^c L$ all forbidden

With our choice of P' parities the couplings at $y=\pi R$ explicitly break SU(5), in the Yukawa and in the gauge-fermion terms. (SU(5) is only recovered in the limit R-> infinity).
But we get acceptable mass terms and can forbid p-decay completely, if desired.

An alternative adopted by Hall&Nomura is to take:
$y=0$: $W_Y= 1/2\ 10 G_u 10 \cdot H_u + 10 G_d 5 \cdot H_d$
$y=\pi R$: $W_Y= -\ 1/2\ 10 G_u 10 \cdot H_u + 10 G_d 5 \cdot H_d$
as if the Yukawa coupling was y-dep. not a constant.

Then, by taking $P'(Q,u^c,d^c,L,e^c)=(+ - + - -)$, SU(5) is fully preserved

One obtains the SU(5) mass relations and p-decay is suppressed but not forbidden.

A different possibility is to put $H^D_{u,d}$ at $y=\pi R/2$ (no triplets) and the matter in the bulk (N=2 SUSY-SU(5) multiplets).

In order to be massless all of them should be ++.
Looks impossible:

PP'	bulk field	mass
++	u^c, e^c, L	$2n/R$
+ -	Q, d^c	$(2n+1)/R$
- +	Q', d'^c	$(2n+1)/R$
- -	u'^c, e'^c, L'	$(2n+2)/R$

(follows from $P=(+++++)$, $P'=(---++)$)

But one can add a duplicate with opposite P':
then we get the full set u^c, e^c, L and Q, d^c at ++

Hebecker,
March-Russell'01

Finally one is free to take some generation in one way, some other in a different way to get flavour hierarchies etc

By using breaking by BC one can stay in 5 dim

S/ZxZ'

Z -> P breaks SUSY
Z' -> P' breaks SO(10) down to G_{PS} = SU(4)xSU(2)$_L$xSU(2)$_R$

(G_{PS} is the residual symmetry on the hidden brane at y=πR/2)

On the visible brane at y=0 SO(10) is broken down to SU(5) (lower rank!) by BC acting as Higgs 16+16$^{\text{bar}}$
(we could use real Higgses localised at y=0 but sending their mass to infinity is more economical)